FRACTURE OF STEEL FIBRE REINFORCED CONCRETE - THE UNIFIED VARIABLE ENGAGEMENT MODEL

by

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Abstract

Cementitious materials are quasi-brittle of nature and have an inherent weakness in resisting in tension. They crack under low levels of tensile stress and usually fail by sudden propagation of these cracks. In order to prevent brittle failure, an appropriate load carrying mechanism must be provided across the crack, such as, steel reinforcement. A similar concept is applicable for the case of fibre reinforced concrete, where discontinuous fibres are added as reinforcement to bridge cracks and to transmit tensile stresses across cracks, thereby improving the performance of the composite structure. Tensile and shear strength are properties of major importance for a wide range of civil engineering materials and structures. One of the major issues with tensile and shear strength studies is the difficulty of determining the strength directly. For example, under shear forces cracks tend to propagate in both Mode I (opening) and Mode II (sliding) configurations.

For engineers to safely design concrete structures or structural members that contain steel fibre reinforcement a simple model is needed, of sufficient accuracy, to describe the basic material laws. In this report, a model named the Unified Variable Engagement Model is developed to describe the behaviour of randomly orientated discontinuous fibre reinforced composites subject to uniaxial tension, shear or mixed-mode fracture. The model is developed by integrating the behaviour of single, randomly oriented, fibres over 3D space and is capable of describing the peak and post-peak response of fibre-cement-based composites under tension and/or shear loading states. Based on quantifiable material and mix parameters, the model is shown to give good predictions when compared to experimental results for various types of fibres for both uniaxial tensile and shear strength tests on fibre reinforced composites.
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NOTATION

\( a \) = a factor
\( a_e \) = a factor
\( a_g \) = maximum size of the aggregate particle
\( b \) = width of a structural element
\( b_e \) = a factor
\( c_1, c_2 \) = coefficients
\( D \) = depth of a structural element
\( d_c \) = distance from the boundary surface in 2D element
\( d_f \) = diameter of fibre
\( E_c \) = modulus of elasticity of cementitious matrix
\( E_f \) = modulus of elasticity of fibre
\( f \) = maximum frictional resistance of fibre through the snubbing zone
\( f_{cm} \) = mean compressive strength of cementitious matrix
\( f_{ct} \) = tensile strength of cementitious matrix
\( f_{cu} \) = cube compressive strength of cementitious matrix
\( f'_c \) = characteristic compressive strength of concrete
\( H_0 \) = elastic constant
\( h \) = height or thickness of a structural element
\( K_b \) = fibre boundary coefficient
\( K_b \) = fibre boundary influence factor
\( K_d \) = damage factor
\( K_f \) = global fibre orientation factor
\( K_{f,2D} \) = global orientation factor in a 2D domain
\( K_{f,3D} \) = global orientation factor in a 3D space
\( K_f \) = global orientation factor
\( K_{fd} \) = fibre dispersion factor
\( k \) = fibre orientation factor
\( k_b \) = a factor
\( k_o \) = a factor accounting for the energy required to deflect a crack around the fibre ends
\( l_a \) = embedment length of fibre
\( l_c \) = critical length of fibre
\( l_f \) = length of fibre
\( l_{hook} \) = total length of hook
\( P \) = applied load/force
\( P_f \) = fibre pullout force
\( P_{f(j)} \) = probability of a fibre crossing a crack
\( P_{dp} \) = maximum double punch failure load
\( P_{f,max} \) = maximum pullout load of fibre
\( P_x \) = fibre pullout force in the direction of the fibre axis
\( Pr(f) \) = probability density function for mix mode fracture
\( Pr(cf) \) = probability density function for effective fibre in mix mode fracture
\( u \) = horizontal or sliding displacement
\( w \) = separation plane opening displacement
\( = \) crack opening displacement
\( w_e, w_{es} \) = crack opening displacement for which the fibre becomes effectively engaged
\( v \) = vertical or opening displacement
\( \alpha \) = engagement constant for UVEM
\( \alpha_f \) = aspect ratio of fibre

\( \alpha_I \) = engagement constant for VEM I

\( \alpha_{II} \) = engagement constant for VEM II

\( \beta \) = unengaged angle

\( \gamma \) = fibre bending angle

\( \gamma_{crit} \) = critical fibre bending angle

\( \gamma_{max} \) = maximum fibre bending angle

\( \varepsilon \) = strain of concrete composite

\( \varepsilon_{cr} \) = strain for which the concrete cracks

\( \theta \) = fibre orientation angle

\( \theta_{crit} \) = critical angle of orientation for fibre engagement

\( \theta_l, \theta_u \) = lower and upper angles for the fibre inclination angle considering the effect of boundary surface in 2D element

\( \theta_{lc}, \theta_{uc} \) = lower and upper angle limits for the fibre inclination angle considering the effect of boundary surface in 2D element

\( \rho_f \) = fibre volumetric content

\( \rho_{f,limit} \) = upper limit of fibre volumetric content

\( \rho_{f, opt} \) = optimum limiting fibre volumetric content

\( \sigma \) = strength of composite

\( \sigma_0 \) = tensile strength of cementitious matrix without fibre reinforcement

\( \sigma_c \) = strength of unreinforced cementitious matrix

\( \sigma_f \) = tensile stress of fibre composite

\( \sigma_{f0} \) = peak tensile strength of fibre composite

\( \sigma_{fu} \) = effective ultimate tensile strength of the fibre

\( \bar{\sigma}_{fu} \) = effective ultimate tensile strength of the fibre

\( \tau_b \) = fibre bond stress
\( \tau_{b,ave} \) = average fibre bond stress

\( \tau_{b,f} \) = contribution of frictional effect to the bond stress

\( \tau_{b,0} \) = initial fibre bond stress

\( \tau_0 \) = shear strength of cementitious matrix without fibre reinforcement

\( \phi \) = angle between applied load and a line normal to the separation plane

\( \phi \) = diameter of cylinder

\( \varphi \) = fibre snubbing angle
CHAPTER 1   INTRODUCTION

In the early 1960's, pioneering research into fibre reinforced concrete was undertaken by Romualdi and Batson (1963) where it was demonstrated that the tensile strength and crack resistance of concrete can be improved by providing suitably arranged, closely spaced, wire reinforcement. After nearly 50 years of research in the development and placement of fibres in reinforced concrete, the concept has matured to the stage where it is finding increasing use in practice. The materials used for fibres have also seen significant advancements including stainless steels and complex polymers. Currently, fibres are used as a form of shear reinforcement in reinforced concrete structural elements, blast resistance in structures, as shotcrete in tunnel linings and slope stabilisation works, limiting early age shrinkage cracking in large concrete pavements, as well as repair and rehabilitation and slab-on-ground (Banthia and Trotter, 1994).

By adding fibres to a concrete mix the objective is to bridge discrete cracks providing for some control to the fracture process and increase the fracture energy. Since the early work of Romualdi and Batson (1963), the pullout mechanism of discontinuous fibres embedded in a variety of cementitious materials has been studied by a number of researchers. Some of the major studies in the field include those of Gray (1984a, b), Gopalaratnam and Shah (1987), Mandel et al. (1987), Namur et al. (1987), Naaman et al. (1989), Namur and Naaman (1989), Wang (1989), Wang et al. (1990a, b), Li (1992) and many others.

The current understanding of the behaviour of fibre-matrix interfacial mechanics is based on a number of pullout studies using single or multiple fibres where steel fibres are embedded within a cementitious matrix. The experimental parameters investigated including the rate of loading (Banthia and Trotter, 1991, Hughes and Fattuhi, 1975 and Maage, 1977), curing and environmental temperatures (Banthia and Trotter, 1989b and Banthia and Trotter, 1992), the quantity and quality of the matrix (Gray and Johnston, 1984, Banthia and Trotter, 1989a, and Gopalaratnam and Abu-Mathkour, 1987), addition of adhesive agents (Guerrero and Naaman, 2000) and fibre type and fibre orientation (Banthia and Trotter, 1994, Naaman and Shah, 1976, and Sorouchian and Bayasi, 1991). In spite of a belief sometimes expressed (Banthia and Trotter, 1994) that no correlation exists between the behaviour of a single fibre pullout test and the
behaviour of bulk fibres in a real composite matrix, the effectiveness of a fibres as a medium of stress transfer is often assessed using fibre pullout tests where slip between the fibres and the matrix is monitored as a function of the applied load.

Despite numerous publications on fibre concrete behaviour, limited research has been undertaken on developing general design models for fibre reinforced composites in tension. Visalvanich and Naaman (1983) derived a semi-empirical model for the tension-softening curve in discontinuous randomly distributed steel fibre-reinforced mortar by assuming a purely frictional fibre-matrix interface and complete fibre pullout. With the same assumptions and taking into account an additional frictional effect called the snubbing effect, Li (1992) derived an analytical model named the fibre pullout model (FPM) that predicts the bridging stress-COD relationship for fibre reinforced brittle-matrix composite. One limitation of this model is it does not account for the potential fracture effect of fibres in the composite.

A micro-mechanical model known as the fibre pullout and rupture model (FPRM) was developed by Maalej et al. (1995). In their model, the FPM model of Li (1992) was extended to account for the possibility of fibre rupture in the composite. The model is able to predict the composite bridging stress-COD relationship, account for fibre pullout, fibre rupture and the local frictional effect or snubbing. A limitation of FPRM is it does not account for interaction between neighbouring fibres and the modification of the matrix properties by the addition of fibres, bending rupture and possible effects of matrix spalling at the exit points of fibres inclined to the cracking plane.

Using the "fibre-matrix misfit" theory of Timoshenko (1941), Naaman et al. (1991a) proposed an analytical model for straight, undeformed, circular steel fibres aligned perpendicularly to the cracking plane. The model was shown to capture the pullout-slip relationship between steel fibres and concrete when compared to the experimental data as published by the authors. However this model is limited for use in directionally orientated plain fibre composites.

Gilles (1999) developed a micro-mechanical model taking into account the different phenomena observed during pullout of a deformed fibre, including the interfacial adhesion between the fibre and the matrix, friction and fibre deformation. The model can be used for predicting pullout behaviour of fibres having various geometries but is limited to the case where the fibres are aligned perpendicular to the cracking plane.
Marti et al. (1999) developed a simple parabolic model to describe the stress-COD relationship of randomly orientated fibre reinforced composites with the tension stress of the fibre composites where it was assumed that after cracking of the matrix there is zero contribution to tensile strength from the matrix and that the shear stress is constant along the shorter embedded length.

Many other models have been proposed such as those of Romualdi and Batson (1963), Aveston and Kelly (1973), Naaman et al. (1973), Pakotiprapha et al. (1974), Gray (1984a), Brandt (1985), Lim et al. (1987) and Easley and Faber (1999) but these models are generally limited in their use as tools for structural designers due to limitations of the models or due to the complexity of the models. In this report, a model named the Unified Variable Engagement Model (UVEM) is developed to describe the behaviour of randomly orientated discontinuous fibre reinforced composites subject to uniaxial tension, shear and mixed-mode fracture. The model, founded on the variable engagement modelling approach of Voo and Foster (2003, 2004) is developed by integrating the behaviour of single, randomly oriented, fibres over 3D space and is capable of describing the peak and post-peak response of fibre-cement-based composites under tension or shear loading states and is based on quantifiable material and mix parameters.

In Chapter 2 of this report the literature is reviewer, indicating the state-of-the-art; in Chapter 3 the UVEM is developed and the model validated in Chapter 4. Finally, in Chapter 5, some conclusions from the study are drawn.
CHAPTER 2  REVIEW OF LITERATURE

2.1 INTRODUCTION

Cementitious materials are generally brittle and have an inherent weakness in resisting tension (Neville and Brooks, 2004). They crack under low levels of tensile stress and usually fail by sudden propagation of these cracks. In order to prevent brittle failure, an appropriate load carrying mechanism must be provided across the crack such as, for example, steel reinforcement.

A similar concept is applicable for the case of fibre reinforced concrete (FRC), where discontinuous fibres are added as reinforcement to bridge cracks and to transmit tensile stress across a crack, thereby improving the performance of the composite structure.

The concept of using fibre reinforcements to improve the tensile characteristic of binding materials dates back to ancient Egyptian times where straw was mixed with mud for masonry construction. Further, the Romans used horse hair to reduce shrinkage in their concrete mixes (Illston, 1996). Research in fibre reinforced concrete was, however, limited until the pioneering research of Romualdi and Batson in early 1960’s, as discussed in Chapter 1.

To-date, several different types of fibres are available, both man-made and natural. The selection of the type of fibre is guided by its mechanical and/or chemical properties as well as the extent that the fibres influence the matrix properties. The fibres commonly used in FRC are often divided into two broad categories (Swamy, 1992):

(i) Low modulus, high elongation fibres such as nylon, polypropylene and polyethylene in which the fibres enhance primarily the energy absorption characteristics only.

(ii) High strength, high modulus, fibres such as steel, glass and asbestos in which the fibres enhance the strength as well as the toughness of the composites.

The application of fibre reinforced composites are varied and dependent on the type of fibres that have been used. Synthetic fibres, such as polyethylene, are used to improve resistance to cracking caused by drying shrinkage. Glass fibres are typically used in production of thin sheet products such as precast architectural panels due to their capability of producing relatively light weight and thin sections. In such an application, the glass fibres act as the primary reinforcement with the fibre content usually in the range of 5% to 15% by volume and special production methods used in manufacturing of the composite (Balaguru and Shah, 1992).
For structural performance, steel fibres have competitive advantages over other fibres due to their high elastic modulus and ability to form strong bond with the surrounding cementitious matrix. In addition, they can be easily deformed and are manufactured in various shapes and patterns, which creates additional mechanical anchorage in concrete. This can significantly enhance the tensile strength and toughness capacities of structural elements.

In this investigation, steel fibres are selected for the development and investigation of the structural behaviour of concrete. The following sections review and discuss the behaviour and constitutive laws of steel fibre reinforced concrete.

2.2 STEEL FIBRE REINFORCED CONCRETE

Steel fibres are commonly made of carbon steel or stainless steel with tensile strengths ranging from 250 to 2300 MPa and modulus of elasticity of 200 GPa. Ideally, in a low tensile strength and brittle cementitious matrix, the matrix will fracture or crack before the steel fibre reaches its ultimate strength (Balaguru and Shah, 1992). Once the matrix has cracked, fibres transmit stresses across the matrix, thereby providing resistance to crack widening. As the crack opens, fibres will be pulled out from the matrix and the fracture toughness of the fibre reinforced composite increases. This fibre pullout process or bond-slip between the fibre and the cementitious matrix plays a critical role that affects the performance of FRC.

A number of steel fibre shapes have been developed by various researchers and manufacturers to improve the mechanical bond characteristics. Some commonly used steel fibre shapes include end-hooked, crimped, corrugated, twin-cone, button ends, indented and twisted polygonal; each having a different resistance mechanism.

During the fibre pullout process, the resistance to crack propagation depends on the bond resistance of the fibre. Three types of fibre matrix bonds are particularly important: chemical adhesion, friction and bond due to mechanical anchorage induced by deformation. Whilst the adhesion bond component is relatively weak, specifically in the case of conventional fibre reinforced concrete where typical fibres used are smooth and effective bond cannot be developed (Maage, 1977), the bond developed through mechanical means (e.g. end-hooks and fibre bending or snubbing) and the frictional bond are significant (Morton and Groves, 1974; Ouyang et al., 1994; Lee and Foster, 2006; Htut and Foster, 2008; Htut, 2010).

From his investigation on fibre bond mechanisms, through discrete fibre pullout tests, Htut (2010) found that the pullout force can be obtained by the summation of its individual mechanical and frictional components. For example, Figures 2.1 and 2.2 show Htut’s plots of load versus separation plane opening displacement for a 35 mm fibre crossing the separation plane
Figure 2.1: Contribution of the hook and frictional effects for 35 mm hooked-end fibres cross at 30 degrees to a separation plane (Htut, 2010).

Figure 2.2: Contribution of the hook and frictional effects for 35 mm hooked-end fibres cross at 60 degrees to a separation plane (Htut, 2010).
at 30 degrees and at 60 degrees, respectively. In each case the curves marked as (1) are the performance of the fibres with the hooks cut-off (i.e. the straight porting of the fibre); the curves marked (2) are greased hooked-end fibres, providing the hook contribution, and the curves marked (3) are the summation of (1) and (2). It was observed that the load versus separation plane opening displacement can be reasonably obtained, within the bounds of experimental variation and uncertainty, by summing each individual component, i.e. the mechanical contribution of the hook and the snubbing and frictional component.

Insight into fibre behaviour during pullout is vital for developing constitutive and design models for FRC and for allowing FRC to be used most effectively and economically. In multi-fibre composites, not all fibres are aligned in the direction of the applied load; instead, almost all fibres lie at various angles to the loading direction. Discrete fibre pullout tests allow observation and understanding of all stages of the pullout process, starting from the initial elastic deformation to fibres orientated at different fibre orientation angles being pulled out or fractured.

Many investigators including Piggott (1974), Morton and Groves (1974), Bentur et al. (1985), Wei et al. (1986), Mandel et al. (1987), Banthia and Trottier (1994), Naaman and Najm (1991), Leung and Ybanez (1997), Shannag et al. (1997), Guerrero and Naaman (2000), Robins et al. (2002), Markovic et al. (2003a), Htut and Foster (2008) and Htut (2010) have carried out discrete fibre pullout tests for Mode I failure. Most of these researchers undertook the individual fibre pullout tests with one end of the fibre embedded in concrete and the other either encased in a protective non-cementitious medium or directly gripped by the clamps of the testing apparatus, as shown in Figure 2.3. In this type of experimental configuration, fibre debonding and pullout occur only on one side of the artificially induced crack, with the fibre bent prior to testing. In contrast, Banthia and Trottier (1994) and Robins et al. (2002) performed single fibre pullout tests with fibres embedded in both sides of the cementitious matrix. However, the single fibre arrangement (see Figure 2.4) provides for a non-symmetric failure, which influences the resulting observations. When the fibre is aligned at an angle to the loading direction, the specimen is subjected to rotation, which affects the pullout load and behaviour.

Morton and Groves (1974) and Htut and Foster (2008) conducted discrete fibre pullout test with fibres embedded in both sides of the matrix with a symmetric fibre arrangement (Figure 2.5). This simulates more realistically the behaviour of fibres as they pullout from the medium. The difference between Morton and Groves (1974) and Htut and Foster (2008) tests is that Morton and Groves used resin as the pullout medium while Htut and Foster (2008) used cement mortar and concrete as the pullout mediums for fibres and used radiographic imaging, with the fibres under load, to witness and measure the fibre-pullout behavioural response.
Figure 2.3: Typical singly fibre pullout test arrangement.

Figure 2.4: Typical Single fibre pullout test with fibre embedded in both side of the matrix (e.g. Banthia and Trottier, 1994, Robins et al. 2002).

Figure 2.5: Oblique discrete fibre pullout tests (e.g. Morton and Groves, 1974, Htut and Foster, 2008).
To the author's knowledge to date, tests on discrete steel fibres in shear have only been conducted by Lee and Foster (2006a, 2006b). Their test arrangement is shown in Figure 2.6.

![Test arrangement for direct shear push-off test](image)

Figure 2.6: Test arrangement for direct shear push-off test Lee and Foster (2006a, b).

2.3 FACTORS INFLUENCING PULLOUT BEHAVIOUR

The pullout behaviour of fibres from cementitious matrices depends on a number of factors including:

- Type of fibres and geometry;
- Fibre length and diameter;
- Fibre orientation with respect to the loading direction;
- Matrix composition and its mechanical properties;
- Fibre volumetric fraction in the composite;
- Specimen preparation method; and
- Rate of loading.

2.3.1 Fibre Geometry

The pullout behaviour of fibres from cementitious matrix is influenced by the fibre geometry. The maximum pullout load is a function of the length of the fibre embedment and the mechanical properties of the binding matrix (Alwan et al., 1999). However, for end hooked steel fibres, plastic deformation of the hooks significantly contributes to the maximum pullout load, and produces higher pullout energy, than for straight fibres, if all other influencing factors are equal.
The hook contribution is dependent on the fibre properties and the hook geometries such as inclination angle, fibre diameter and hook length. In addition, by isolating the frictional component of bond from its mechanical bond, Naaman and Najm (1991) and Htut (2010) noted that the mechanical component of the bond is responsible for the improvement in the maximum pullout load.

Waved shape fibres or indented fibres usually have a more non-linear load crack opening displacement (COD) response than that of end hooked steel fibres and, consequently, more energy is required to pull out the fibres. However, this type of fibre is best suited to a low strength matrix, as in a high-strength matrix excessively deformation of the fibres may cause fracture of the fibre in the process of pullout, leading to a considerable reduction in the pullout energy (Banthia, 1990).

Torex fibres or twisted fibres with triangular cross section have been reported to have a better pullout response and a significant improvement in ductility over other fibre types (Naaman, 2000). Sujivorakul and Naaman (2003) reported that although the maximum pullout load of Torex and end-hooked steel fibres is almost the same, the pseudo-plastic behaviour in which the high level of pullout load is maintained for a slip of about 70% to 90% of the fibre embedded length is only observed in the Torex fibre. For this reason, the pullout work and pullout energy of Torex fibres are approximately 80% to 130% higher than for end hooked steel fibres (Sujivorakul and Naaman, 2003), as shown in Figure 2.7.

![Comparison of typical tensile stress versus slip response of smooth, hooked and twisted steel fibres under pull-put (Naaman, 2003).](image)

Figure 2.7: Comparison of typical tensile stress versus slip response of smooth, hooked and twisted steel fibres under pull-put (Naaman, 2003).
2.3.2 Fibre Embedment Length and Diameter

The fibre aspect ratio, which is the ratio of embedded fibre length over the fibre diameter, can also influence the fibre pullout behaviour. For the same fibre geometry, larger fibre embedment and/or diameter means that a larger surface area of fibre is in contact with the cementitious matrix. The greater contact area suggests that larger pullout load and pullout energy are expected. The experimental evidence suggests, however, that this only holds true for straight steel fibres (Gopalaratnam and Abu-Mathkour, 1987) and is not usually the case for the end hooked steel fibre (Naaman and Najm, 1991; Alwan et al., 1999; Markovic et al., 2003b). Robins et al. (2002) argued that the pullout response of end hooked fibres is influenced by its embedment length, fibre orientation and matrix strength. They also reported that the toughness and magnitude of the pullout responses are proportional to the fibre initial length of embedment. Similar findings were reported on the importance of fibre embedment length on the pullout performance of corrugated, or waved form, fibres by Chanvillard and Aïtcin (1996).

2.3.3 Fibre Inclination Angle

In fibre reinforced concrete containing randomly distributed fibres, not all fibres are aligned in the direction of the applied load. The fibre inclined with the load direction means that the fibre has to realign to the direction of the applied load. As a result of the fibre inclination, the fibre will experience a combination of shear, bending and tensile stresses.

Due to the fibre inclination angle, fibres are found to be bent at their exit point, i.e. at the crack surface, and snubbing or spalling of the matrix at the exit point is expected for a fibre with high inclination angle (Morton and Groves, 1974). Therefore, in addition to frictional pullout resistance, the pulley effect and the bending mechanism primarily account for the increased value of maximum pullout load (Ouyang et al., 1994). As illustrated in Figure 2.8, only the component in the direction of the fibre axis $P_x$ of the total pullout force $P$ is dedicated to the pullout of an inclined fibre, and may be referred as the pulley effect (Ouyang et al., 1994).

Naaman and Shah (1976) suggested that spalling, or snubbing, of matrices due to fibre inclination angle leads to a substantial reduction in pullout resistance. On the other hand, Lee and Foster (2006a) and Htut (2010) argued that snubbing contributes a significant mechanical anchorage. Snubbing usually occurs for fibres at high fibre inclination angles where the matrix locally spalls due to fibre abrasion at the point of entry. Fibres that subjected to snubbing action undergo significant bending in addition to axial tension due to pullout and, consequently, the pullout load is likely to maintain a longer plateau compared to that of aligned fibres.
The bending resistance of fibres depends on the angle of inclination of the fibre, type of fibre, fibre material properties and the quality of the surrounding cementitious matrix. Figures 2.9 and 2.10 show the relationship between fibre inclination angle and the maximum pullout force achieved for single straight and end-hooked steel fibres, respectively. The values of the maximum pullout load are given relatively to the maximum pullout load of a fibre aligned with respect to the loading direction.

Figure 2.9: Effect of the inclination angle on the pullout capacity of a single straight fibre embedded in a cementitious matrix.
Figure 2.10: Effect of the inclination angle on the tensile load-bearing capacity of a single end hooked fibre.

At a high fibre inclination angle of 60°, Figure 2.9 shows that straight fibres carry significant load. This is mainly due to the significant increase in mechanical anchorage due to the snubbing effect as the fibre orientation angle increases.

At high inclination angles of 60°, there is only a small difference of pullout force when comparing to aligned fibres. Maage (1977a) argued that the low shear and tensile strength of the matrix are the governing factors after the angle passes a certain value, depending on the material properties. In addition, the mechanical properties of the fibre become more influential as the fibre orientation angle increases (Robins et al., 2002) and, consequently, a high fibre inclination angle may cause a fibre to fracture due to the high stress concentration as the result of bending (Markovic et al., 2003b). However, the slips at maximum pullout load were usually significantly greater for inclined fibres than for aligned fibres.

Local crushing and spalling of the matrix during the pullout process is influenced by the work undertaken in the fracture process, and measured as the area under the pullout load versus displacement or slip curve. Figure 2.11 shows the relationship between fibre pullout energy and fibre inclination angle for tests on single end-hooked steel fibres. The influence of fibre orientation on the fracture (or pullout) energy is clearly demonstrated with a maximum toughness value occurring for fibre orientations between approximately 0° and 20°. Similar findings on improvement in pullout energy, with respect to initial increasing fibre inclination angle beyond 0°, was
Figure 2.11: Effect of inclination angle on the pullout energies of a single end hooked fibre.

reported by Naaman and Shah (1976), Banthia and Trottier (1994), Robins et al. (2002) and Htut (2010). Beyond a critical inclination angle (approximately 30$^\circ$) the pullout energy reduces.

In the research of Naaman and Najm (1991), Robins et al. (2002) and Htut (2010), some end hooked steel fibres fractured during the pullout process due to stress concentration. In their experimental works, Robins et al. (2002) reported that fibre fracture is likely to occur when the fibre embedment length is greater than 5 mm and the fibre inclination angle is greater than 20$^\circ$. According to them, the fibre fracture is caused by the relatively high stress concentration in the fibre at the bending point (i.e. near the crack surface). When fibre fractures, the pullout load will drop suddenly and the pullout behaviour is similar to that of a smooth steel fibre and, consequently, significant reductions in pullout energies are expected. In some cases, fracture of the matrix (snubbing) around the fibre bending point, due to its bearing against the matrix, was also observed.

Htut (2010) provided some insights of the snubbing effect in Mode I fracture using radiographic X-ray imaging. Using the imaging analysis, Htut measured the snubbing lengths of fibre for different fibre inclination angles (Figure 2.12). According to Htut (2010), the effect and magnitude of fibre snubbing can be measured indirectly from the fibre snubbing angles, $\phi$. Figure 2.13 shows the plot of measured normalised fibre snubbing angles against the fibre inclination angles for various separation plane opening displacements. The fibre snubbing angle is determined as a function of the separation plane opening displacement (SPOD), $w$, and fibre inclination angle, $\theta$, and given as:
Figure 2.12: Plot of snubbing length over fibre diameter at peak load versus fibre angles (Htut, 2010).

Figure 2.13: Plot of normalised fibre snubbing angles versus fibre inclination angles for various separation plane opening displacement (SPOD) (Htut, 2010).
\[ \varphi = \alpha \theta / (a \theta + b) \]  

(2.1)

where \( \alpha_f = l_f' / d_f, l_f' = \) length of fibre, \( d_f = \) diameter of the fibre, \( a = -0.22w \) and \( b = 20w + 70. \)

Figure 2.14 shows the peak load versus fibre inclination angle for the Mode II fracture data of Lee and Foster (2006a, b) for end-hooked fibres of length \( l_f' = 60 \text{ mm} \) by diameter \( d_f = 0.9 \text{ mm}, \) 1200 MPa tensile strength, and straight fibres of \( l_f' = 48 \text{ mm}, \) with the straight fibre produced by cutting off the hooks of end-hooked fibres. Based on their experimental observations, Lee and Foster argued that the load versus shear displacement behaviour of the discrete fibre push-off specimen is dependent on the orientation of the fibre across the shear plane. They noted that the peak load occurs at a relatively small shear displacement for both end-hooked and straight fibres when the orientation angle is inclined towards the loading direction (positive angles in Figure 2.14) while for fibres aligned against the direction of the loading (negative angles), considerable displacement occurs before the fibres become effectively engaged. This is as a result of the influence of snubbing. They confirmed that the critical length of a fibre for fibre fracture is influenced significantly by the angle that the fibre crosses the cracking plane and that the combined influence of bending and axial action has a pronounced influence on fibre fracture.

In Figure 2.15, the movement of the separation plane required for a fibre to be taken as engaged \((w_{es})\), defined by Lee and Foster as the displacement corresponding to the point on the ascending load-slip curve where the force in the fibre is equal to one-half of its pullout strength, is a function of the angle that the fibre crosses the separation plane. The Figure shows that only those fibres at high positive angles are likely to be effective in carrying load over the engineering range of a few millimetres. That is, half or more of the fibres do not carry load efficiently. Subsequently, Lee and Foster (2006a, b) concluded that the fibre orientation angle and snubbing effect dominate the behaviour of Mode II fracture of SFRC.

### 2.3.4 Matrix composition and its mechanical properties

In ordinary Portland cement concrete, the presence of fine cementitious materials such as finely grounded cement, silica fume, fly ash, metakaolin, slag and latex can improve the particle packing of the matrix and improve the interfacial microstructure of concrete. This can, subsequently, improve the frictional bond strength and increase debonding energy (Mandel et al., 1987; Naaman and Najm, 1991; Najm et al., 1994; Banthia, 1990; Banthia et al., 1995; Banthia and Yan, 1996; Guerrero and Naaman, 2000; Markovic et al., 2003b).

The use of very fine sand in cementitious matrices generally leads to improvement in the pullout behaviour of smooth, end-hooked and Torex steel fibres (Guerrero and Naaman, 2000). In comparison with normal sand (approximately 7 mm in particles size) and fine sand cementitious matrices, an increase in pullout energies of more than 100% was reported when smooth
Figure 2.14: Peak load versus fibre angle (Lee and Foster, 2007, 2008).

Figure 2.15: Fibre engagement ratio versus fibre angle of orientation (Lee and Foster, 2008).
steel fibres, in which the frictional component of bond is dominant, are embedded in fine sand cementitious matrix. For end-hooked and twisted fibres, the increase in pullout energy was more than 50%. Similarly, an improvement in the maximum pullout force, up to approximately 150% for smooth steel fibres and approximately 50% for end-hooked steel fibres, was reported by Guerrero and Naaman (2000). Since the debonding process between the steel fibres and the matrix is localised within the interfacial transitional zone around the fibre, the aggregate composition and its fineness have an indirect influence on the fibre pullout response. In addition, by compacting the FRC, the bond between the fibre and the matrix is improved and, thus, improves the pullout response (Pinchin and Tabor, 1978). However, the presence of coarse aggregate in the FRC will affect the compaction effort and the weaker fibre matrix interfacial zone will be formed and, consequently, influences the pullout load.

Naaman and Najm (1991) stated that the bond characteristic between fibre and the matrix is very complex because of the combined actions of several bond components, with one of them being the compressive strength of the matrix. Their experimental results showed that higher bond strength can be observed between the fibre and the matrix as the compressive strength of the matrix increases. Similar behaviour was also observed by Banthia and Trottier (1994) with experiments undertaken with normal, mid and high strength concrete.

Valle and Büyükoztürk (1993), who investigated the Mode II fracture of steel FRC, suggested that the pullout behaviour of fibre is greatly dependent on the matrix strength. Similarly, Khaloo and Kim (1997) also investigated the effect of compressive strength of the matrix on the direct shear behaviour of FRC. Khaloo and Kim (1997) concluded that an increase of concrete strength will result in higher bond developed at the fibre/concrete interface and, consequently, FRC specimens have a higher ultimate shear stress than their plain concrete counterparts.

2.3.5 Fibre volumetric fraction in the composite

In a typical pullout test, usually only a single fibre is pulled out from the cementitious matrix. In reality, however, fibres are more closely spaced in fibre reinforced composites. Naaman and Najm (1991) and Markovic et al. (2003b) suggested that the presence of additional fibres around another fibre acts as a form of secondary reinforcement and provides an improvement in the fibre pullout response. They postulated that adding shorter fibres together with longer fibres into a fibre cocktail can improve the response due to lateral confinement provided by the short or micro-fibres. In addition, the short or micro-fibres also bridge the secondary micro-cracks that develop around the fibre hook during its plastic deformation (Markovic et al., 2003b). Microfibres can also improve the peak and the post-peak strength for small crack opening displacements, while macrofibres is efficient for large crack opening displacements. Therefore, hybrid combinations of short and long steel fibres can improve concrete toughness for both small and large crack opening displacements (Sorelli et al., 2005).
Naaman and Shah (1976), Maage (1977a) and Mandel et al. (1987) reported that for aligned fibres, increasing the number of fibres from the same pullout medium does not significantly affect the mean pullout load per fibre. On the other hand, Naaman and Shah (1976) argued that for fibres inclined at 60°, the efficiency of fibre bond is inversely proportional to the number of fibres being pullout from the same area. Their reasoning was that it may be related to the disruption of the matrix in the pullout region with the increasing number of fibres.

In multi-fibre composite, Li et al. (1998) reported that the improvement in tensile performance can be achieved by incorporating a high fibre volume fraction, especially in the case of steel fibres. Similarly, in Mode II fracture, Khaloo and Kim (1997) and Lee (2007) observed that the shear strength of FRC is increased as fibre proportion is increased. Higher steel fibre volume in the concrete matrix resulted in larger shear stress regardless of concrete strength and fibre aspect ratio. However, in fibre reinforced mortar, Htut (2010) demonstrated that there is an upper limit to the optimum fibre content. He inferred that if the fibre content is higher than the limiting optimum content, failure will occur between the fibres and in the concrete (through crushing), which is undesirable, and that this limit is more likely be observed in the case of very high strength fibres being pulled through a weak matrix.

Figure 2.16 shows the interaction between fibres and the matrix during the fibre pullout process. According to Htut (2010), the upper limit of fibre volume (or optimum fibre volume) to ensure that a failure plane passes through the fibres, and not around the fibre ends can be determined as:

\[ \rho_{f, opt} \leq \frac{2k_o f_{cf}}{\tau_b \alpha_f} \]  

(2.2)

where \( k_o \) is a factor to account for the energy required to deflect the crack around the fibre ends, \( f_{cf} \) is the tensile strength of the matrix, \( \tau_b \) is the average fibre bond stress and \( \alpha_f \) is the aspect ratio of the fibre.

### 2.3.6 Specimen preparation method

Based on the various pullout experiments that have been undertaken, it can be seen that a degree of contradiction exists in the literature. It is hypothesised that this is due to the natural variability between different concrete mixes as well as various methods in specimen preparation and environmental test conditions.

It has been reported that different method of specimen preparation, such as casting the specimens at different direction and different curing time have substantial influences on the bond strength (Gray and Johnston, 1978, 1984; Banthia and Trottier, 1989). Testing undertaken in various environmental conditions, such as temperature and humidity will affect the bond strength of FRC (Banthia and Trottier, 1989 & 1992; Wongtanakitcharoen and Naaman, 2004).
Figure 2.16: Fibre-matrix interaction showing: (a) potential failure surfaces, (b) fibre pullout failure, (c) matrix fracture around the fibre end and (d) multi-fibres interaction (Htut, 2010).

Ferrara et al. (2010) found that careful design of casting procedure and a suitable balanced set of fresh state properties can have favourable effect on the dispersion and the orientation of fibres in concrete. In their study, four point bending test was used as the performance indicator of FRC cast with different procedures. The first slab was cast by pouring the FRC at one short edge of the moulds, and allowing the FRC to flow parallel to the long sides. For the second slab, FRC was poured centrally along one long edge and this allowed the fresh FRC to spread radially. The experiment gave favourable result to the first slab. By casting the FRC in the direction aligned to tensile stress, so will be the fibres, the structural element will display a superior structural performance such as deflection hardening or even strain hardening behaviour, but will have little residual tensile strength in the direction normal to the direction of the aligned fibres.

2.3.7 Rate of Loading

For straight steel fibres, Gokoz and Naaman (1981) and Baathia (1990) observed the frictional bond after the initial slip is insensitive to the loading velocity or rate of pullout. This contradicts
the findings of Burakiewicz (1978). Gokoz and Naaman (1981) also reported that pullout energy was independent of loading velocities.

For end-hooked steel fibres, Dong Joo et al. (2008) reported that the loading rate has an insignificant effect on the peak pullout load, and pullout energy, of the single fibre from a matrix. On the other hand, Banthia (1990) found that a pullout rate of 2.12 mm/s results in an increase in the maximum pullout force, as well as pullout energies, of approximately 10 to 40%, when compared with the lower pullout rate of 8.46 m/s. However, changes in the rate of pullout also caused changes in the mode of failure and, consequently, fibre fracture in some cases may result in a brittle failure of FRC. In addition, sustained strains can reduce the pullout resistance of deformed fibres (Banthia, 1990).

For elements with straight fibres subjected to impact loading, a peak pullout load of up to 4.6 times greater than the static loading was reported by Banthia and Trottier (1991). They also reported that a greater pullout energy was achieved for impact than for static pullout forces, provided that a pullout failure mode is maintained and the fibres do not fracture.

2.4 CONSTITUTIVE BEHAVIOUR OF SFRC

There have been numerous analytical models that have been developed to model bond in steel fibres being pulled from cementitious matrices. Two different approaches to the fibre debonding model have been developed and these analyses are classified into strength-based approaches and fracture-based approaches. In the strength-based approach, Leung and Li (1991) stated that debonding occurs when the shear stress at the interface reaches the interfacial shear strength. In the fracture-based approach, Stang et al. (1990) argued that the debonded region is treated as a crack that will propagate if a critical interfacial fracture toughness, which usually expressed in terms of a critical energy release rate, is overcome at the crack tip.

Aveston et al. (1971) introduced an analytical model based on the energy principle, where the micromechanical parameters are used to calculate the energy release rate independently for each fracture process during the multiple cracking process. Aveston and Kelly (1973) further improved the model of Aveston et al. (1971) by introducing bond between the fibre and the matrix as well as taking into consideration the alignment of the fibre relative to the loading direction.

Tjiptobroto and Hansen (1993) extended the energy based approach of Aveston et al. (1971) for high performance cement composites with discontinuous fibres. The model assumed that multiple cracks form subsequently one after another up to a certain saturation level. Their model predicts that the existence of multiple microcracking depends on the fibre properties as well as
on the interface and matrix properties such as debonding energy, the interface frictional stress and matrix strength.

Based on the shear-lag theory, Grezczuk (1969) derived the shear stress distribution along the fibre length with the assumption that elastic matrix and complete fibre-matrix debonding takes place when the maximum interfacial shear stress is equal to the maximum interfacial bond shear stress. Lawrence (1972) assumed that partial debonding between fibre and matrix occurs where debonding is initiated at the entrance point of fibre to matrix and propagates to the end of the fibre. In his analysis, Lawrence took frictional resistance acting over the debonded portion of the fibre into account in the load transfer process.

Visalvanich and Naaman (1983) derived a semi-empirical model for the tension-softening curve in discontinuous randomly distributed steel fibre-reinforced mortar by assuming a purely frictional fibre-matrix interface and complete fibre pullout. With the same assumptions, and taking into account an additional frictional effect called the snubbing effect, Li (1992) derived an analytical model named the fibre pullout model (FPM) that predicts the complete bridging stress-COD relationship for a fibre reinforced brittle-matrix composite. One limitation of this model, however, is it does not account for the potential fracture effect of fibres in the composite.

A micromechanical model known as the fibre pullout and rupture model (FPRM) was developed by Maalej et al. (1995). In their model, the FPM model of Li (1992) was extended to account for the possibility of fibre rupture in the composite. The model is able to predict the composite bridging stress-COD relationship, accounting for fibre pullout, fibre rupture and the local frictional effect or snubbing. A limitation of FPRM is it does not account for interaction between neighbouring fibres and the modification of the matrix properties by the addition of fibres, bending rupture and possible effects of matrix spalling at the exit points of fibres inclined to the cracking plane.

Naaman et al. (1991) proposed an analytical model for straight, undeformed, circular steel fibres aligned perpendicularly to the cracking plane using the Timoshenko (1941) "fibre-matrix misfit" theory. The model was shown to capture the pullout-slip relationship between steel fibres and concrete when compared to the experimental data as published by the authors. However, this model is limited for use in directionally orientated plain fibre composites.

Li and Chan (1994) argued that the debonding criterion for steel fibres in a cementitious matrix is governed by a strength-based approach. Based on single fibre pullout tests on straight steel fibres, they found that frictional stress is the dominant bond property and it would be adequate to describe the interfacial debonding in a composite model. They argued that the debonding mode, namely strength-based or fracture-based, is not related to the location of interfacial debonding.
Chanvillard (1999) developed a micromechanical model taking into account the different phenomena observed during pullout of a deformed fibre, including the interfacial adhesion between the fibre and the matrix, friction and fibre deformation. The model can be used for predicting pullout behaviour of fibres having various geometries but is limited to the case where the fibres are aligned perpendicular to the cracking plane.

However, the majority of these models are generally limited in their use as tools by structural designers due to limitations of the models or due to the complexity of the models. For design, engineers require a simple yet reliable approach that explains the mode of fracture and models, with sufficient accuracy, the behaviour of steel fibre reinforced concrete under various loading conditions.

Marti et al. (1999) developed a simple model to describe the stress-COD relationship of randomly orientated fibre reinforced composites with the tension stress of the fibre composites, $\sigma_f$, given by

$$\sigma_f = \sigma_{f0} \left( 1 - \frac{2w}{l_f} \right)^2$$  \hspace{1cm} (2.3)

where $\sigma_{f0}$ is the peak tensile strength, $w$ is the crack opening displacement (COD) and $l_f$ is the length of the fibres. The peak tensile strength is, in turn, given by

$$\sigma_{f0} = \frac{\rho_f \ell_f \tau_b}{2d_f}$$  \hspace{1cm} (2.4)

where $\rho_f$ is the volumetric fraction of fibres, $d_f$ is the diameter of the fibres and $\tau_b$ is the bond stress between the fibres and the concrete. In Equation 2.3, it is assumed that after cracking of the matrix there is zero contribution to tensile strength from the matrix and that the shear stress, $\tau_b$, is constant along the shorter embedded length.

Voo and Foster (2003, 2004) introduced Variable Engagement Model (VEM) for Mode I fracture of FRC. The Variable Engagement Model considers the slip between the fibres and the matrix occurs before the full bond stress is developed and also includes the condition where the fibres can fracture before being pulled out across a crack. Subsequently, Lee and Foster (2008) extended the uniform bond stress based approach of Voo and Foster (2003, 2004) for the Mode II fracture of FRC based on a lump bond stress approach, and is termed as Variable Engagement Model II (VEMII). A full review of the different versions of the VEM, and the approach can be found in Voo and Foster (2003, 2004), Lee and Foster (2007) and Voo and Foster (2009).
2.5 SUMMARY

Fibre reinforced concretes have emerged as a subject of major interest among researchers and structural engineers for their ease of application and potential for construction cost efficiencies. While many fibre types and materials are reported in the literature, for strength limit state conditions steel fibres have been largely preferred over other fibre materials due to their higher elastic modulus and their ability to form a good bond with the surrounding cementitious matrix. Steel fibres improve the mechanical properties of quasi-brittle, cementitious, materials such as ductility, durability, energy absorption, fatigue, and toughness. Generally, the resistance to crack propagation depends on the bond resistance of the fibre that, in turn, depends on the material properties of the cementitious matrix and the fibre properties such as the geometry, orientation and aspect ratio.

A number of analytical models have been developed to describe bond in steel fibres being pulled from cementitious matrices. However, many of these models are limited in their use as tools for structural design due to their complexity or limit of application. The exception to this are the models of Marti et al. (1999) and Voo and Foster (2003, 2004) that describe the physically behaviour and, yet, are relatively simple for use in the design office.

The review of the literature highlights many of the physical effects that influence the behaviour of fibres as they pullout across a crack or discontinuity. For example the influence of any hook or end effect or, indeed, its sectional shape and longitudinal geometry must be considered in the development of a rational, physical, model. Effects such as ‘snubbing’ have also been shown to particularly influential on behaviour of SFRC in many situations, and this is particularly so for the case of straight fibres that are orientated in the matrix, and cross the fracture plane, at an angle to the loading direction. For any physical model to be successful, the model must be representative of such observations.
CHAPTER 3 VARIABLE ENGAGEMENT MODEL FOR MIXED MODE FRACTURE OF SFRC

3.1 INTRODUCTION

A number of analytical models have been developed to describe the constitutive behaviour of fibre reinforced concrete. However, as outlined in Chapter 2, many of these models are complex and/or are regression approaches and, hence, hinder them from engineering design application. Engineers require a simple and reliable model, which explains the fracture processes of the fibre reinforced concrete. The variable engagement model (VEM) first developed by Voo and Foster (2003, 2004) being an example of one such model.

Voo and Foster (2003, 2004) introduced the Variable Engagement Model for Mode I fracture of fibre reinforced concrete (FRC) and for consistency of notation we shall refer to this model as the VEMI (as noted in Voo and Foster, 2009). The VEMI considers the slip between the fibres and the matrix that occurs before the full bond stress is developed and also includes the condition where the fibres can fracture before being pulled out across a crack. In that model, a uniform bond stress-slip relationship was adopted. Lee and Foster (2008) extended the uniform bond stress based approach (VEMI) of Voo and Foster (2003, 2004) for Mode II fracture of FRC based on both uniform and lump bond stress approaches, and is termed as Variable Engagement Model II (VEMII). A full review of the different versions of the VEM, and the approach can be found in Voo and Foster (2003, 2004, 2009), Foster et al. (2006), Lee and Foster (2008) and Foster and Voo (2009). In this study the variable engagement modelling approach is extended to mixed-mode fracture, unifying the VEMI and VEMII models. The model is thus named the unified variable engagement model, or UVEM, and its development is reported herein.

To date, fibres in the various variable engagement models have been assumed to be uniformly dispersed in three dimension and the wall effects have been assumed as small. Both VEMI and VEMII were developed to model the Mode I and II fracture behaviour of FRC in three dimen-
sions (3D). Generally, when steel fibre is uniformly dispersed in an infinitely large volume of concrete, the fibres are expected to be randomly oriented with equal probabilities of being oriented in any direction in the space. However, with the presence of boundaries, the fibres located near the boundaries are orientated somewhere between three and two dimensions (3D and 2D). Further, structural elements made of high performance fibre reinforced concrete are often very thin (Simon et al., 2002) and fibres near boundaries, especially long ones, will be biased towards a 2D plane. In this unified model, the boundary (wall) effect is also included.

3.2 UNIFIED VARIABLE ENGAGEMENT MODEL

3.2.1 General

It is well established that for quasi-brittle materials loaded in tension and/or shear, such as concrete, that, after cracking, localisation dominates the behaviour. Before cracking the behaviour can be expressed in terms of stress versus strain such as that shown in Figure 3.1, where $H_0$ is the elastic constant (for uniaxial tension, $H_0$ is the elastic modulus, for shear it is the shear modulus). After cracking, the localized behaviour is described by the load versus crack opening, or separation plane displacement, $w$, as shown in Figure 3.1.

![Diagram showing stress versus strain/displacement.](image)

Figure 3.1: Stress versus strain/displacement.
In the model presented in this study and that of the other VEM models, the strength behaviour of a fibre reinforced composite, $\sigma$, is obtained by a summation of its individual components; that is, the overall response of FRC is due to the combination strength of the un-reinforced matrix, $\sigma_c$, and the strength contribution of each individual fibre crossing the failure plane, $\sigma_f$. That is:

$$\sigma = \sigma_c + \sigma_f \quad (3.1)$$

In the development of the UVEM model, the following assumptions are made:

(i) Fibres centred at more than one-half a fibre length away from a boundary. The geometric centres of the fibres are uniformly dispersed in space and all fibres have an equal probability of being oriented in any direction;

(ii) Fibres centred at less than one-half a fibre length from a boundary are influenced by wall effects;

(iii) Fibres that pull out do so from the side of the crack with the shorter embedded length while the longer side of the fibre remains rigidly embedded in the matrix;

(iv) Displacements due to elastic strains taking place within the fibres are small in comparison to the displacements arising from movement occurring between the fibres and the matrix; and

(v) The energy expended by bending of fibres compared to that of pullout of the fibres is small and can be neglected.

### 3.2.2 Matrix Component

For unreinforced mortar and concrete, the tension softening stress can be taken as (Voo and Foster, 2004, 2009; Lee and Foster, 2007, 2008; Htut and Foster, 2010):

$$\sigma_c = c_1 \sigma_0 e^{-c_2 w} \quad (3.2)$$

where $\sigma_0$ is the strength of the concrete without fibre reinforcement and $c_1$ and $c_2$ are coefficients. In Equation 3.2, if the angle between the applied load and an arbitrary line normal to the separation plane (or crack surface) is $\phi$ and $\gamma$ is the angle between the applied load and inclined discrete fibre, as illustrated in Figure 3.2, then when $\phi = 0^\circ$, $\sigma_0$ is equal to tensile strength of the concrete. On the other hand, when $\phi = 90^\circ$, $\sigma_0$ represents the shear strength of the concrete, $\tau_0$. 

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Figure 3.2: Discrete fibre orientation and definition of fibre bending angle, $\gamma$ (Htut and Foster, 2010; Htut, 2010).

In Equation 3.2, the coefficient $c_1$ accounts for the beneficial effect of the fibres on the peak matrix strength while $c_2$ is a factor that controls the steepness of the descending branch and is also influenced by the volume of fibres, the cementitious matrix composition, as well as the strength of the unreinforced matrix. In assessment of the uniaxial tensile strength of a cementitious matrix, Voo and Foster (2003, 2004), Htut and Foster (2010), Htut (2010) and Lee et al. (2011) adopted $c_1$ as unity. In contrast, $c_1$ is taken as $1 + 72\rho_f$ for Mode II fracture (Lee and Foster, 2007, 2008; Htut and Foster, 2010; and Htut, 2010). The $c_2$ coefficient is dependent on the fibre volume concentration in the composite. Htut (2010) observed that an increase in fibre volume concentration resulting an increase in matrix spalling and the crack is likely to propagate around or near the ends of the fibres. Thus, he considered the material coefficient $c_2$ to be inversely proportional to the fibre volume concentration. Voo and Foster (2003, 2004, 2009) used $c_2 = 15$ for both concrete and mortar while $c_2$ is equal to 15 and 30 for concrete and mortar, respectively, as adopted by the Lee et al. (2011b) study.

3.2.3 Fibres Component

The relationship between a fibre and the direction of pullout relative to the matrix, as depicted in Figure 3.2. The fibre orientation angle, $\theta$, is the angle between the fibre and a line drawn normal to the crack or fracture plane. A clockwise direction is taken as positive, whereas an anticlockwise direction is negative. The fibre bending angle, $\gamma$, and maximum fibre bending angle, $\gamma_{max}$, are then defined by:
\[ \gamma = |\theta - \phi| \quad \text{for} \quad 0 \leq \gamma \leq \pi \] \hspace{1cm} (3.3)

\[ \gamma_{\text{max}} = |\phi| + \pi/2 \quad \text{for} \quad \pi/2 \leq \gamma \leq \pi \] \hspace{1cm} (3.4)

where the loading direction angle is \( \phi = \tan^{-1}(u/v) \). For Mode I fracture, \( \phi = 0 \) and \( \gamma_{\text{max}} = \pi/2 \). For Mode II fracture, \( |\phi| = \pi/2 \) and \( \gamma_{\text{max}} = \pi \). The crack separation displacement \( (w) \) is defined as:

\[ w^2 = u^2 + v^2 \] \hspace{1cm} (3.5)

where \( u \) and \( v \) are the Mode II (sliding) and Mode I (opening) components, respectively.

### 3.2.4 Engagement Model

In FRC, an individual fibre crossing a crack is either engaged or unengaged, where an engaged fibre carries load and an unengaged one does not. Before a fibre is engaged effectively, a small opening at the crack may first occur due to effects such as slip of the fibre in the matrix and/or snubbing at the fibre-matrix interface. This is evident from the tests of Lee and Foster (2006), Htut and Foster (2007), Lee (2007), Lee and Foster (2007), Htut and Foster (2008), Ng et al. (2010) and Htut (2010).

Fibres that cross at angles close to \( \theta = 0^\circ \) engage quickly while fibres at large angles engage much later. In the various versions of the VEMs (Voo and Foster, 2004, 2009; Foster et al., 2006; Lee and Foster, 2008), the form of the engagement model adopted comes from the single fibre pullout test data.

In VEMI, the crack opening displacement, \( w_e \), for which the fibre becomes effectively engaged in the load carrying mechanism is termed the engagement length and is denoted as \( w_e \). Assuming the engagement length versus fibre bond slip can be described using a continuous function then the boundary conditions dictate that for \( \theta = 0^\circ \), \( w_e = 0 \) and the function was taken as asymptotic to \( \theta = \pi/2 \). The function that was adopted by Voo and Foster (2003, 2004) can be written as (Voo and Foster, 2009):

\[ w_e = \alpha_f/f \tan \theta \] \hspace{1cm} (3.6)

where \( \alpha_f \) is the engagement parameter and is based on data obtained from discrete fibre pullout tests. In the determination of the fibre engagement function, a fibre is taken to be engaged at
the point where the force in the fibre is 50% of its peak load value. Based on their previous works, Voo and Foster (2009) suggested that the engagement parameter can be taken as:

\[ \alpha_f = \frac{1}{3.5 \alpha_f} \]  

(3.7)

where \( \alpha_f \) is the aspect ratio of the fibre and is taken as \( l_f / d_f \), in which \( l_f \) is the fibre length and \( d_f \) is the fibre diameter. The engagement angle at which fibres become active is then determined as:

\[ \theta_{crit} = \tan^{-1} \left( 3.5 \frac{w}{\alpha_f l_f} \right) \]  

(3.8)

where \( \theta_{crit} \) is the point where fibres oriented at \( \theta \leq \theta_{crit} \) carry load while for fibres at \( \theta > \theta_{crit} \) are yet to be engaged.

Based on the discrete fibre tests subjected to Mode II action, Lee and Foster (2006, 2007) proposed the fibre engagement function for Mode II fracture as:

\[ w_e = \alpha_f l_f \tan^2 \left( \frac{\pi}{2} - \frac{\theta}{2} \right) \]  

(3.9)

where \( \alpha_f = 1 / 2 \alpha_f \). The critical angle for which fibres are becoming active, \( \theta_{crit} \) can thus be obtained as:

\[ \theta_{crit II} = \pi - 2 \tan^{-1} \left( \frac{w_e}{\alpha_f l_f} \right) \]  

(3.10)

In developing the UVEH model, Htut and Foster (2010) found that using the engagement function proposed by Voo and Foster (2003, 2004, 2009), the fibre engaged much later and that fibres with large \( \theta \) continue to contribute significantly to the composite strength, even for large \( w \). However, this was not reflected in the observation from their single fibre pullout test experimental results. In contrast, the Lee and Foster’s (2006, 2007) VEMII engagement function is observed to engage earlier compared to that of the experimental data.

With additional experimental data reported in Htut (2010) and Ng (2011), the following unified fibre engagement function is proposed:
\[ w_e = \alpha l_f \tan^2 \left( \frac{\gamma}{2} \right) \]  

(3.11)

where \( \alpha = 3/(2\alpha_f) \). The critical angle for which fibres are becoming active, \( \gamma_{crit} \), can hence be obtained as:

\[ \gamma_{crit} = 2 \tan^{-1} \left( \frac{w_e}{\eta \alpha l_f} \right) \]  

(3.12)

The data of the discrete fibre pullout tests of Lee and Foster (2006, 2007) for Mode II, Htut and Foster (2007, 2008) and Htut (2010) for Mode I, Htut (2010) for Mixed Mode in cement mortar and Ng (2011) for Modes I and II fracture in geopolymer concrete are plotted in Figure 3.3. Also plotted is Equation 3.12 and is shown correlate sufficiently well with all the experimental data set. Based on Equation 3.12, in Mode I fracture, the fibre orientated at \( \pi/2 \) will be theoretically engaged at \( \alpha l_f \). However, in reality, this is not possible due to the high bending stiffness of the fibre causing the matrix to spall at fibre inclination angles that lie close to \( \pi/2 \). This effect will be accounted in assessing the probability of an effectively engaged fibre crossing the fracture plane, which is presented in the following Section.

Figure 3.3: Comparison of all discrete fibre pullout tests in this study, mix mode fracture data and that of Lee and Foster (2007) and Htut (2010) against Equation 3.12.
3.3 STRESS VERSUS CRACK OPENING DISPLACEMENT

3.3.1 General

In the VEM, there are five possible states that a fibre can be in. These are:

(i) the fibre is not engaged and may enhance the strength of the matrix when engagement is activated;

(ii) the fibre is fractured upon engagement and, therefore, does not contribute to the strength of the matrix;

(iii) the fibres is at small angle to the cracking plane and does not contribute to the strength of the matrix as the surrounding concrete spalls;

(iv) the fibre was engaged but has since pulled out from the matrix and no longer adds to the strength of the matrix; or

(v) the fibre is engaged and contributing to the strength of the matrix.

A fibre will fracture if the fibre has an embedment length greater than its critical length, $l_c$, such that the force due to the bond between the fibre and the cementitious matrix is greater than the ultimate tensile capacity of the fibre or the fibre is subjected to significant bending in addition to the axial tension. The later, however, only occurs at large bending angles with respect to the fracture plane and its importance is diminished as these fibres do not engage until a large crack opening and/or sliding has occurred (Voo and Foster, 2003, 2004). Fibres at large angles usually pull-out before their engagement and do not contribute to the strength.

Taking the maximum embedded length of a fibres as $l_{a,max} = l_f/2$, the critical length of fibres such that fibre fracture need not be considered in the design model can be written as:

$$l_f < l_c = \frac{d_f}{2} \times \frac{\bar{f}_{fu}}{\tau_b}$$

where $\bar{f}_{fu}$ is an effective ultimate tensile strength of the fibre, considering fibre bending, and $\tau_b$ is the average bond stress along the fibre length.

Various models for including fibre fracture with bending in the VEM approach for calculating the $\sigma$–$w$ relationship are presented in Voo and Foster (2004, 2009) and Lee and Foster (2007). The models, however, are complex and before adding this additional level of complexity, the impact of fibre fracture and bending on the resulting $\sigma$–$w$ curve needs first to be assessed.
Looking at the fibre fracture conditions, it is observed that fibre fracture is more likely for the higher bending angled fibres and these fibres do not engage until large separation displacements first occur. In the VEM approach, fibres at the large angles will pullout before engagement and do not contribute to the strength. Voo and Foster (2004, 2009) took the effective stress of the fibre for consideration of fibre fracture as $\bar{\sigma}_{fu} = \sigma_{fu}$, while for Mode II fracture Lee and Foster (2007, 2008) recommended $\bar{\sigma}_{fu} = 0.5\sigma_{fu}$. For the case of mixed mode fracture, it is suggested that fibre fracture need not be considered if Eq. (6.16) is satisfied with:

$$\bar{\sigma}_{fu} = \sigma_{fu} \times \frac{\pi}{2\gamma_{max}}$$  \hspace{1cm} (3.14)

If the inequality of Equation 3.13 is violated then a significant portion of the fibres may fracture and fibre fracture should be considered in calculating the $\sigma-w$ curve. The formulation for the case where fibre fracture is and is not critical are presented below in Sections 3.3.2 and 3.3.3, respectively.

### 3.3.2 Stress versus Crack Opening Displacement Excluding Fibre Fracture

When a single fibre is pulled out from a matrix, the force, $P_f$, in the fibre is:

$$P_f = k\pi d_f \tau_b l_a$$  \hspace{1cm} (3.15)

where $k$ is fibre orientation factor and is:

$$k = 0 \quad \text{for} \ w < w_e \ \text{and} \ w > l_a$$ \hspace{1cm} (3.16a)

$$k = 2(l_a - w)/l_f \quad \text{for} \ w_e < w \leq l_a$$ \hspace{1cm} (3.16b)

In Equation 3.15, $l_a$ is the length of embedment of the fibre and $\tau_b$ is the average shear stress between the fibre and the matrix. Marti et al. (1999) noted that for $w = 0$ the average length of embedment is $l_f/4$ and that the number of bonded fibres decreases linearly with increasing $w$.

For fibres randomly orientated in two or three dimensions, Equation 3.15 is integrated over a plane of unit area in a 3D space or a 2D domain and one obtains the following stress:

$$\sigma_f = K_fK_{fd}\alpha_f\rho_f\tau_b$$  \hspace{1cm} (3.17)

where $K_f$ is the global orientation factor, $K_{fd}$ is the fibre dispersion factor, $\rho_f$ is the fibre volumetric ratio and $\tau_b$ is the average shear stress for all engaged fibres.
The fibre dispersion factor, $K_{fd}$, accounts for the uniformity and the fibre dispersion along the crack path and is a function of the fibre volume, fibre length and the particle size distribution in the cementitious matrix. By observing the fibre pullout behaviour in the uniaxial tension specimens using X-ray radiography, Htut (2010) undertook a statistical survey on distribution of the fibres and the fracture process of the fibre reinforced cement mortar. He confirmed that the cracks were initiated and propagated along the path with minimum resistance or at locations with poor fibre dispersion where a crack location is not constrained by geometry or by an induced weakness, such as a saw cut. For 25 mm long fibres dispersed in cement mortar matrix, Htut (2010) determined that $K_{fd} = 0.82$, whereas if a crack is constrained in its initial location, for example with the cutting of a notch, $K_{fd} = 1$. In this study, $K_{fd}$ is taken as unity for all cases.

3.3.3 Fibre Orientation Factor, $K_f$

The fibre orientation factor, $K_f$ is determined by probability of the fibre crossing the fracture plane and is affected by the shape of the domain. Figure 3.4 shows the orientation of a fibre crossing a 2D space. The probability density function of a fibre, the $K_f$ factor, can be derived as:

$$K_f = \frac{\int_{-\pi/2}^{\pi/2} \cos \theta \, d\theta}{\int_{-\pi/2}^{\pi/2} \, d\theta} = \frac{2}{\pi}$$

(3.18)

![Diagram](image)

Figure 3.4: Polar coordinates and averaging for orientation in two dimensional (2D) space.
When a fibre is crossing a plane in 3D space, various values of the $K_f$ factor have been proposed in the literature and are listed in Table 3.1. In Table 3.1, Martí et al. (1999) and Voo and Foster (2003, 2004) considered the randomness of the fibre embedment length. A fibre crossing a crack is assumed to have an initial embedment length on the shorter side of between zero and $l_f/2$, with the embedment length of the fibre decreasing linearly from its initial value to zero when $w = l_a$. Moreover, with reducing fibre embedment length with increasing $w$, the proportion of bonded fibres also decreases linearly and the reduction of the fibre bonded strength can be factored by $1 - 2w/l_f$. For the case of mixed mode fracture we then write:

$$K_f = \left(\frac{a + b}{\pi}\right) \cdot \frac{1}{2} \left(1 - \frac{2w}{l_f}\right)^2$$  \hspace{1cm} (3.19)

where the first term $(a + b)/\pi$ is the probability density function of a fibre crossing the crack. The $a$ and $b$ values, as illustrated in Figure 3.5, are:

$$a = \min\left[\gamma_{\text{crit}}, \frac{\pi}{2} + |\phi|\right]$$  \hspace{1cm} (3.20a)

$$b = \gamma_{\text{crit}}$$  \hspace{1cm} (3.20b)

![Figure 3.5: Polar coordinates and averaging for orientation for mix mode fibre fracture in two dimensional (2D) space.](image-url)
Table 3.1: Summary of 3D fibre orientation factors for fibres subjected to tension action.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>( K_f )</th>
<th>3D Factor</th>
</tr>
</thead>
</table>
| Romualdi & Mandel | 1964    | \[
\frac{\int_{0}^{\pi/2} \int_{0}^{\pi/2} \cos \phi \cos \psi d\phi d\psi}{\int_{0}^{\pi/2} \int_{0}^{\pi/2} d\phi d\psi} \]
\[
\frac{\pi/2 \pi/2}{\pi/2 \pi/2} \]
\[
\int_{0}^{\pi/2} \int_{0}^{\pi/2} d\phi d\psi \]
|               |          | 0.405                                          |           |
| Parimi et al.   | 1973    | \[
\frac{\int_{0}^{\pi/2} \cos \theta d\theta}{\int_{0}^{\pi/2} d\theta} \]
\[
\int_{0}^{\pi/2} \cos \theta d\theta \]
|               |          | 0.637                                          |           |
| Aveston & Kelly | 1973    | \[
\int_{0}^{\pi/2} \cos \theta \sin \theta d\theta \]
\[
\int_{0}^{\pi/2} \cos \theta \sin \theta d\theta \]
|               |          | 0.5                                            |           |
| Pakotiprapha    | 1976    | \[
\frac{\int_{0}^{\psi \phi} \cos ^2 \phi \cos ^2 \psi d\phi d\psi}{\int_{0}^{\psi \phi} d\phi d\psi} \]
\[
\int_{0}^{\psi \phi} \cos ^2 \phi \cos ^2 \psi d\phi d\psi \]
|               |          | 0.25                                           |           |
| Marti et al.    | 1999    | \[
\frac{1}{2} \left( 1 - \frac{2w}{l_f} \right)^2 \]
\[
\frac{1}{2} \left( 1 - \frac{2w}{l_f} \right)^2 \]
|               |          | variable                                       |           |
| Foster          | 2001    | \[
\int_{0}^{\pi/3} \cos \theta \sin \theta d\theta \]
\[
\int_{0}^{\pi/3} \cos \theta \sin \theta d\theta \]
|               |          | 0.375                                          |           |
| Voo & Foster    | 2003    | \[
\frac{1}{\tan\left( \frac{w}{3.5 \alpha \mu_f} \right)} \left( 1 - \frac{2w}{l_f} \right)^2 \]
\[
\frac{1}{\tan\left( \frac{w}{3.5 \alpha \mu_f} \right)} \left( 1 - \frac{2w}{l_f} \right)^2 \]
|               | 2004    | variable                                       |           |
| Stroeven and Hu | 2006    | \[
\frac{\int_{0}^{\pi/2} \cos \theta \sin \theta d\theta}{\int_{0}^{\pi/2} \sin \theta d\theta} \]
\[
\int_{0}^{\pi/2} \cos \theta \sin \theta d\theta \]
\[
\int_{0}^{\pi/2} \sin \theta d\theta \]
|               |          | 0.5                                            |           |
| Stroeven        | 2009 & 2010 | \[
\frac{\pi/2}{\pi/2} \]
\[
\int_{0}^{\pi/2} \cos \theta \sin \theta d\theta \]
\[
\int_{0}^{\pi/2} \sin \theta d\theta \]
|               |          | 0.5                                            |           |
3.3.4 Fibre Orientation Factor in a 2D Domain, $K_{f,2D}$

In a 2D domain, the fibre that is close to the direction of the applied load will be the first to be engaged. Hence, the probability of a fibre crossing the crack, $P_f$, is:

$$P_f = \cos(\gamma - \phi)$$  \hspace{1cm} (3.21)

Integrating Equation 3.21 over the 2D domain, the probability density function for mix mode fracture, $Pr_f$, is:

$$Pr_f = \frac{1}{\pi} \left\{ \int_{0}^{\pi/2 + \phi} \cos(\gamma - \phi) d\gamma + \int_{\pi/2 + \phi}^{\pi/2 + \phi + \beta} \cos(\gamma - \phi) d\gamma \right\}$$  \hspace{1cm} (3.22)

Observations of the test data show that fibres at very acute angles to the crack plane do not pull out from the matrix but rather rip out from the matrix. Thus, only fibres orientated at angle more than some minimum angle $\beta$ to the fracture plane are considered as being effective (Figure 3.6). The probability density function, $Pr_{(\phi)}$, can be determined as:

$$Pr_{(\phi)} = \frac{1}{\pi} \left\{ \int_{0}^{\phi} \cos(\gamma - \phi) d\gamma + \int_{\phi}^{\pi/2 + \phi - \beta} \cos(\gamma - \phi) d\gamma \right\}$$  \hspace{1cm} (3.23)

Figure 3.6: Region of effective fibre for mix mode fibre fracture in two dimensional (2D) plane.

37
\[ a_e = \min \left[ -\frac{\pi}{2} + \phi + \beta, 0 \right] \]  
\[ b_e = \max \left[ -\frac{\pi}{2} + \phi + \beta, 0 \right] \]  

(3.24a)
(3.24b)

Moreover, the probability density function for mix mode fracture accounting for the randomness of the fibre embedment length, between zero to \( l_f/2 \) can be determined as:

\[
Pr_{(ef;l_a)} = \frac{2}{l_f} \int_{0}^{l_f/2} \left\{ \frac{1}{\pi} \left( \int_{0}^{a_e} \cos(\gamma - \phi) d\gamma + \int_{b_e}^{\pi/2 + \phi - \beta} \cos(\gamma - \phi) d\gamma \right) \right\} d\gamma \times \left( 1 - \frac{2w}{l_f} \right) 
\]  

(3.25)

Finally, with the inclusion of the factor to account for the proportion of bonded fibres, which decreases with increasing \( w \), the \( K_f \) factor in a 2D domain is:

\[
K_f = \frac{\pi K_b}{4} \frac{l_f/2 - w}{l_f} \left\{ \frac{1}{\pi} \left( \int_{0}^{a_e} \cos(\gamma - \phi) d\gamma + \int_{b_e}^{\pi/2 + \phi - \beta} \cos(\gamma - \phi) d\gamma \right) \right\} d\gamma \times \left( 1 - \frac{2w}{l_f} \right) 
\]  

(3.26)

where \( K_b \) is the fibre boundary influence factor, which will be described in detail in the following section. Note that for 2D, \( \pi K_b / 4 = 1 \).

In Substituting \( \gamma = \gamma_{crit} \), that is for the point of engagement corresponding to particular crack opening displacement \( w \), into Equation 3.26, the relationship between critical fibre engagement angle \( \gamma_{crit} \) and the orientation factor for mixed-mode fracture may be obtained, as shown for the two-dimensional case plotted in Figure 3.7.

For Mode I fracture in a 2D domain (\( \phi = 0 \)), \( K_f \) simplifies to:

\[
K_{f,2D} = \frac{2 \sin \gamma_{crit}}{\pi} \left( 1 - \frac{2w}{l_f} \right)^2 \quad \text{..... for } \gamma_{crit} \leq \frac{\pi}{2} - \beta
\]  

(3.27)

while for Mode II fracture in a 2D domain (\( \phi = \pi/2 \)), \( K_f \) is given by:

\[
K_{f,2D} = \frac{1 - \cos \gamma_{crit}}{\pi} \left( 1 - \frac{2w}{l_f} \right)^2 \quad \text{..... for } \beta \leq \gamma_{crit} \leq \pi - \beta
\]  

(3.28)

In Equations 3.27 and 3.28, if \( \gamma_{crit} \) is outside of the limit then \( K_{f,2D} \) calculated with \( \gamma_{crit} \) taken at the limit.
Figure 3.7: Relationship between critical fibre engagement angle $\gamma_{crit}$ and the two-dimensional orientation factor, $K_{f,2D}$, for various mixed-mode fracture angles $\phi$.

A comparison of the proposed $K_f$ factors for Modes I and II fracture in a 2D plane is presented in Figure 3.8 and it is observed that, for a given crack opening displacement, the values of $K_f$ for Mode II are lower than that of Mode I. This is because more fibres in Mode II engage later than in Mode I.

### 3.3.5 Fibre-Boundary Influence Factor, $K_h$

In terms of three dimensions, the presence of a boundary restricts a fibre from being freely oriented and limits the fibre angle. This is referred to as the boundary effect and is depicted in Figure 3.9.

Based on the work of Romualdi and Mandel (1964), Soroushian and Lee (1990) were the first to develop a $K_f$ formulation that included the boundary effect. However, the formulation has been shown as incorrect (Stroeven, 2010). Lee et al. (2011) presented a new formulation for $K_f$ with consideration given to the effect of the member dimension. At the boundary, the out of plane fibre inclination is not possible and will therefore be oriented in a 2D plane while far away from the boundary, the fibre is freely orientated in the 3D space. This means that the $K_f$ factor varies between $2/\pi$ in a 2D plane and 0.5 in a 3D space. In developing their $K_f$ factor, Lee et al.
(2011a) assumed that the crack surface is perpendicular to the boundary surface and the fibre orientation can be in one of the following three situations:

(i) The fibre orientation is affected by both long and short parts of the fibre;

(ii) The fibre orientation is affected by only the longer part; and

(iii) The fibre orientation is not affected by the length of the fibre.

Figure 3.8: Proposed 2D fibre orientation factor.

Figure 3.9: The effect of boundary surface on fibre inclination angle (Lee et al., 2011a).
According to Lee et al. (2011a), the possible angle of fibre orientation in a member with one side of boundary can be calculated as:

\[
\theta_f(l_a) = \sin^{-1}\left[ \min\left(1, \frac{d_c}{l_a}\right) \right]
\]

(3.29)

\[
\theta_u(l_a) = \sin^{-1}\left[ \min\left(1, \frac{d_c}{l_f - l_a}\right) \right]
\]

(3.30)

and the limiting angle of fibre orientation in a member with one side of boundary is:

\[
\theta_{lc}(l_a, \theta) = \sin^{-1}\left[ \min\left(1, \frac{\sin \theta}{\sin \theta}\right) \right]
\]

(3.31)

\[
\theta_{uc}(l_a, \theta) = \sin^{-1}\left[ \min\left(1, \frac{\sin \theta_u}{\sin \theta}\right) \right]
\]

(3.32)

The orientation factor, \( K_f \), considering the influence of member thickness in a 2D member, as graphically presented in Figure 3.10, can be expressed by:

\[
K_{f,2D} = \frac{2}{l_f} \int_0^{l_f/2} \frac{\pi/2}{\pi} \int_0^{\pi/2} \cos \theta \cdot \left[ \theta_u(l_a, \theta) + \theta_{lc}(l_a, \theta) \right] \cdot \sin \theta d\theta \\
\]

(3.33)

\[
= \frac{2}{l_f} \int_0^{l_f/2} \left[ \theta_u(l_a, \theta) + \theta_{lc}(l_a, \theta) \right] \cdot \sin \theta d\theta
\]

Figure 3.11 shows the effect of the member thickness on the \( K_f \) factor in cases with two boundaries. Lee et al. (2011) also presented a derivation for \( K_f \) that accounts for 3D members with a rectangular cross-section (i.e. with four boundaries). Where the height of a beam is relatively large to that of its thickness, the effect of the member thickness is more influential than the effect of member height. In the model developed herein only the thickness effect is considered.

In Equation 3.33, the expression in \{ \} is defined as the fibre-boundary influence factor, \( \bar{K}_b \), and can be written as:

\[
\bar{K}_b = \frac{\pi/2}{\pi} \int_0^{\pi/2} \cos \theta \cdot \left[ \theta_u(l_a, \theta) + \theta_{lc}(l_a, \theta) \right] \cdot \sin \theta d\theta
\]

(3.34)

\[
= \frac{\pi/2}{\pi} \int_0^{\pi/2} \left[ \theta_u(l_a, \theta) + \theta_{lc}(l_a, \theta) \right] \cdot \sin \theta d\theta
\]
Figure 3.10: Surface are on sphere representing the fibre angle (Lee et al. 2011a).

Figure 3.11: Variation of fibre orientation factor in 2D element (Lee et al. 2011b).
The $\bar{K}_b$ factor ranges from 1 for a fully 2D boundary to $\pi/4$ for 3D. The effect of member thickness on $K_f$ can be obtained by integrating through the thickness of the element. This is done by including the coefficient $\bar{K}_b$ in Equation 3.26, as shown in Figure 3.12, and the boundary coefficient is then written as:

$$K_b = \frac{4}{\pi} \int_{h} \bar{K}_b dh$$  \hspace{1cm} (3.35)

where $h$ is the thickness of the element.

![Graph showing fibre boundary influence factor, $K_b$.](image)

Figure 3.12: Fibre boundary influence factor, $K_b$.

Thus, for the fully 3D case, $K_b = 1$, while for the case of 2D $K_b = 4/\pi$.

The value of $K_b$ may be solved by numerical integration. Alternatively, $K_b$ can be obtained with good accuracy from:

$$K_b \approx \frac{4}{\pi} \left( 1 - 0.15 \left[ 0.5 + \frac{h/l_f - 1}{1 + |h/l_f - 1|} \right] \right) \geq 1$$  \hspace{1cm} (3.36)

From Figure 3.12, it can be seen that the effect of the member thickness on $K_b$ is particularly significant when the member thickness is less than twice the fibre length.
3.3.6 Fibre Orientation Factor in a 3D Domain

For Mode I fracture in a 3D domain, the fibre orientation factor, \( K_f \) can be simplified to:

\[
K_{f,3D} = \frac{1}{2} \sin \gamma_{\text{crit}} \left( 1 - \frac{2w}{l_f} \right)^2 \quad \text{for} \quad \gamma_{\text{crit}} \leq \frac{\pi}{2} - \beta
\]  

(3.37)

while for Mode II fracture in a 3D domain, \( K_f \) is given by:

\[
K_{f,3D} = \frac{1 - \cos \gamma_{\text{crit}}}{4} \left( 1 - \frac{2w}{l_f} \right)^2 \quad \text{for} \quad \beta \leq \gamma_{\text{crit}} \leq \pi - \beta
\]  

(3.38)

A comparison of the proposed \( K_f \)-factors for Modes I and II fracture in a 3D domain is presented in Figure 3.13.

![Figure 3.13: Proposed 3D fibre orientation factor.](image)

3.3.7 Bond Stress

The force in a single aligned fibre, \( P_f \) is:

\[
P_f = 0 \quad \text{..... for} \quad w < w_c \quad \text{and} \quad w \geq l_a
\]

(3.39a)

\[
P_f = \pi d_f^2 v_b (l_a - w) \quad \text{..... for} \quad w < w_c \leq w < l_a
\]

(3.39b)
VEMI assumed a constant mean shear stress along the fibre for a given fibre matrix structure, while VEMII discretised the shear contributions of the fibre into three distinct components; the hooked, snubbing and straight frictional zones (Figure 3.14). With the uniform bond approach, the effects of each component (straight, snubbing and hook) are smeared along the remaining embedded length of the fibre. The smeared bond stress, $\tau_b$ is calculated as:

$$\tau_b = \frac{P_{f,\text{max}}}{\pi d_f l_a}$$

(3.40)

where $P_{f,\text{max}}$ is the maximum force in the fibre and $l_a$ is taken as the embedded length of the fibre at the point of engagement. That is at $l_a - w_e$.

Figure 3.14: (a) Fibre performance zones and (b) shear contributions.
In the UVEM we adopt a uniform bond modelling approach but adjust the bond strength according to the angle of the fibre to account for the increasing influence of mechanically induced friction through the snubbing zone. Based on the data of Htut (2010), the following expression is proposed:

\[ \tau_b = \tau_{b,0} + 0.25\gamma^3 \]  

(3.41)

where \( \tau_{b,0} \) is the initial bond stress. The average bond stress \( \tau_{b,ave} \) for all engaged fibres is then obtained from:

\[ \tau_{b,ave} = \frac{1}{\gamma_{crit}^3} \int_0^{\gamma_{crit}} \tau_b d\gamma = \tau_{b,0} + 0.0625\gamma_{crit}^3 \]  

(3.42)

Based on the experimental investigation (Htut, 2010), the initial fibre bond strength, \( \tau_{b,0} \), was found to be:

\[ \tau_{b,0} = 0.8 \sqrt{f_{cm}} \]  

for end hooked fibres  

(3.43a)

\[ \tau_{b,0} = 0.4 \sqrt{f_{cm}} \]  

for straight fibres  

(3.43b)

Referring to Figure 3.15, if the maximum frictional resistance of the fibre through the snubbing zone is \( f \), the contribution of frictional effect to the bond stress, \( \tau_{b,f} \) can be calculated as:

\[ \tau_{b,f} = f \cdot \left( \frac{\gamma}{2} \right) \]  

(3.44)

![Figure 3.15: Snubbing and frictional effect when the fibre is pulled out.](image)
Subsequently, the average bond stress $\tau_{b.\text{ave}}$ for all engaged fibres can thus be obtained from:

$$\tau_{b.\text{ave}} = \tau_{b.0} + \frac{1}{\gamma_{\text{crit}}} \int_{0}^{\gamma_{\text{crit}}} \tau_{b.f} d\gamma = \tau_{b.0} + f \cdot \left[ 1 - \frac{2}{\gamma_{\text{crit}}} \sin \left( \frac{\gamma_{\text{crit}}}{2} \right) \right]$$  \hspace{1cm} (3.45)

Taking $f$ as 4.5 MPa provides good correlation to the fibre bond strength Mode I data of Htut (2010) and Mode II data of Lee (2006) for their 60 mm end-hooked and 48 mm straight fibre data, as seen in Figure 3.16.

![Figure 3.16: Fibre bond stress relationship given by Equation 3.44 and compared with Modes I and II test data of Htut (2010) and Lee (2006), respectively.](image)

3.3.8 Stress versus Crack Opening Displacement Including Fibre Fracture

Using the model described by Equation 3.17 above, any arbitrarily orientated fibre will fracture if:

$$l_a \geq \frac{d_f}{2} \times \frac{\bar{\sigma}_{fu}}{\tau_{b.\text{ave}}} + w_e$$  \hspace{1cm} (3.46)

where $\bar{\sigma}_{fu}$ and $\tau_{b.\text{ave}}$ are given by Equations 3.14 and 3.44, respectively.
For a given crack opening displacement, the global orientation factor is given by:

\[
K_f = \frac{K_b}{l_f(l_f - 2w)} \int_0^{l_{a,\text{crit}} - w} \left\{ a_e \cos(\gamma - \phi) d\gamma + b_e \cos(\gamma - \phi) d\gamma \right\} \times \max(l_{a,\text{crit}} - w, 0) dl_a \cdot \left(1 - \frac{2w}{l_f}\right)
\]

(3.47)

where:

\[
l_{a,\text{crit}} = \min\left(\frac{l_c}{2} + w_e, \frac{l_f}{2}\right)
\]

(3.48)

Equation 3.47 may be solved by numerical integration. For the case of \(l_c < l_f\) no fibres fracture and Equation 3.47 reduces to that of Equation 3.26.

### 3.3.9 Comments on the Unengaged Fibre Region Angle, \(\beta\)

Fibres orientated close to the cracking plane may not be possible to be engaged due to the bending stiffness of the fibre causing the matrix to spall, as shown in the uniaxial tension tests of Htut (2010) and Ng (2011). In this regard, the influence of fibres at low angles relative to the cracking plane may be included directly into the fibre orientation factor, \(K_f\), through the integration limits. This replaces the damage factor, \(K_d\), that was suggested by Voo and Foster (2003).

A similar concept was proposed by Foster (2001) but with constant angle of \(\beta = 30\) degrees. From observations of the uniaxial tension tests in Htut (2010), the angle \(\beta\) appears to depend on the fibre volumetric ratio, types of the matrix (mortar and concrete), strength properties of the matrix, modular ratio of the fibres relative to that of matrix \(E_f / E_c\), type of fibre (end hooked and straight) and the fibre properties (length and diameter). It may be hypothesised that:

(i) the higher the fibre volumetric content, higher the propensity of fibres to cluster together thus increasing the value of \(\beta\);

(ii) the larger the fibre diameter, the greater the fictional contact between the fibre and the matrix and this increases the snubbing area and the angle \(\beta\);

(iii) the angle \(\beta\) is larger when the matrix strength is lower and/or the fibre has higher stiffness;
(iv) the higher the $E_f/E_c$ ratio, the higher the relative stiffness of the fibres and, hence, more work is required in bending of the fibre and increases the $\beta$ region;

(v) the end hook anchorages in the fibre increase the snubbing effort and more work is required for pulling out the fibre. Thus, the $\beta$ angle increases.

There is, however, insufficient test data to validate these hypotheses and, thus, they remain open at this time. Figure 3.17 presents the influence of various $\beta$ angles on the $K_f$ for Mode I fracture in a 3D domain. From the figure, it is observed that the effect is minimal when $\beta \leq 30$ degrees.

In the context of this study, for conventional strength matrix, the angle of $\beta$ is assumed to increase linearly with the fibre volumetric content, $\rho_f$, as shown in Figure 3.18. As mortar is less stiff than concrete, one would expect that the $\beta$ angle is higher for mortar matrix than for concrete. The limiting maximum angle of $\beta$ for end hooked fibres in mortar and concrete is taken as 30 degrees, as adopted by Foster (2001). For straight fibres, less snubbing effort and lesser energy are required to pull out the fibres; hence, the damage to matrix is less and the angle of $\beta$ is taken as half that of angle of $\beta$ for end hooked fibres (i.e. 15 degrees).

![Figure 3.17: Influence of angle $\beta$ on the fibre orientation factor, $K_f$.](image-url)
3.3.10 Comments on the Fibre Volume Content, $\rho_f$, in Unified Variable Engagement Model

In order to discount the ineffectiveness of high fibre volumes in contributing the tensile strength of FRC, the fibre volume content in UVEM model is taken as:

$$\rho_f \leq \rho_{f,\text{limit}}$$

(3.49)

where $\rho_{f,\text{limit}}$ is the limiting maximum fibre volume content. Beyond the limit it is assumed that additional fibre content does not further improve the tensile performance.

In the Htut (2010) experiments, the uniaxial tension specimens were cast using 40 MPa nominal compressive strength cement mortar with end hooked fibres. The fibres were 0.3 mm in diameter, had a length of 25 mm and a tensile strength of 2300 MPa. Four series of tests with fibre volumetric content of 0.5%, 1.0%, 1.5% and 2.0% were carried out. The experimental observation demonstrated that with increasing the fibre volume content, more energy is required to deform the end hooks sufficiently to pull them out of the matrix. However, due to the higher energy required, cracks propagate around or near the ends of end hooked fibres and the fibres
become less effective. The net result is a tearing fracture across the element, rather than one based on fibre pullout. Interestingly, all the specimens behaved similarly and had similar fracture energies (Figure 3.19). Hence, for Htut (2010)'s tests, the limiting maximum fibre volume content, $\rho_{f, limit}$, is estimated to be approximately 0.5%.

In contrast, Suseyto (2010), who tested uniaxial tension specimens using end hooked fibres in concrete with a 50 MPa nominal compressive strength, did not observed this behaviour. The fracture energy of the uniaxial tension specimens increase with fibre volumetric content even up to 1.5%, as shown in Figure 3.20. The fibres that Suseyto (2010) used were 0.62 mm in diameter, had a length of 50 mm and a tensile strength of 1050 MPa.

Further, for cement mortar reinforced with 30 mm long by 0.3 mm in diameter straight steel fibres, the data of Petersson (1980) does not show a limiting maximum fibre volume content for specimens with 0.25% to 1.0% by volume of fibres (Figure 3.21). Again, the fracture energy of the uniaxial tension specimens increase proportionally with fibre volumetric content.

![Fracture energy versus fibre volume concentration](image)

Figure 3.19: Fracture energy versus fibre volume concentration for Htut (2010)'s uniaxial tension tests.
Figure 3.20: Fracture energy versus fibre volume concentration for Suseyto (2010)’s uniaxial tension tests.

Figure 3.21: Fracture energy versus fibre volume concentration for Petersson (1980)’s uniaxial tension tests.
Mode II fracture tests were carried out by Lee and Foster (2006) with 35 mm long by 0.55 mm diameter end-hooked fibres and 13 mm long by 0.2 mm diameter straight fibres in 40 MPa cement mortar. The fibre volumetric ratios investigated were $\rho_f = 0.5\%, 1.0\%, 1.5\%$ and 2%. The fracture energy increases proportionally with fibre volumetric content (Figure 3.22) and, again, a limiting maximum fibre volume content is not noticed.

Based on the experimental evidence from various investigators, it is not possible to define a general model for the limiting maximum fibre content. Additional tests with different fibre properties, fibre volumetric contents and fibre types with different types of cementitious matrix with various strength grades are needed to understand the behaviour. For the purpose of this study, the limiting maximum fibre content, where possible, is determined from the experimental fracture energy data.

![Figure 3.22: Fracture energy versus fibre volume concentration for Lee and Foster (2006)'s direct shear tests.](image)

3.3.11 Summary

Limited research has been undertaken on developing general design models for fibre reinforced composites under mixed mode fracture. In this study, the Unified Variable Engagement Model
(UVEM) has been developed to describe the stress versus crack opening displacement for a fibre reinforced concrete composite subjected to mixed mode fracture. The model, developed from the variable engagement approach of Voo and Foster (2003, 2004) is developed by integrating the effects of individual fibres crossing a cracking plane, together with the plain concrete component. In the development of the unified model, the results and observations of discrete fibre tests of Lee and Foster (2007), Htut (2010) and Ng (2011) were used. The validation of the UVEM model for Mode I and Mode II tests from the literature for concrete and mortar is presented in the following chapter.
CHAPTER 4  VERIFICATION OF THE UVEM

4.1 INTRODUCTION

In this Chapter, the unified variable engagement model (UVEM) developed in Chapter 3 is validated against the experimental data on steel fibre concrete obtained from the literature. In the verification process that follows, the initial bond shear strength, \( \tau_{b,0} \), is taken as:

\[
\tau_{b,0} = k_b \sqrt{f_{cm}}
\]  

(4.1)

where \( f_{cm} \) is the mean compressive cylinder strength and \( k_b \) is given in Table 4.1.

<table>
<thead>
<tr>
<th>Fibre Type</th>
<th>Matrix Type</th>
<th>( k_b )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>End hooked</td>
<td>Mortar</td>
<td>0.67</td>
<td>Voo and Foster (2003)</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>0.8</td>
<td>Foster (2010)</td>
</tr>
<tr>
<td></td>
<td>Reactive powder</td>
<td>1.0</td>
<td>Voo et al. (2010)</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>0.3</td>
<td>Voo and Foster (2003)</td>
</tr>
<tr>
<td></td>
<td>Reactive powder</td>
<td>0.4</td>
<td>Foster (2010)</td>
</tr>
<tr>
<td></td>
<td>Reactive powder</td>
<td>0.6</td>
<td>Voo et al. (2010)</td>
</tr>
</tbody>
</table>

Where no compressive strength data was available but tensile strength was provided by the investigators, the initial bond strength is taken as:

\[
\tau_{b,0} = 3k_b f_{ct}
\]  

(4.2)

where \( f_{ct} \) is mean tensile strength of the unreinforced cementitious matrix.

Where the initial bond stress, \( \tau_{b,0} \), was measured directly by the investigators by using discrete fibre pullout tests, the results obtained using this measured value is also plotted in the presented
figures. Comparisons are made between the results obtained using bond strength as predicted using the matrix strength properties with those obtained from the measured value.

Equation 3.2 was used to compute the stress and the crack opening displacement (COD) relationship for the unreinforced cementitious matrix with \( c_f = 1 \) for Mode I fracture and \( c_f = 1 + 72 \rho_f \) for Mode II fracture, as discussed in Section 3.2. Aggregate size plays an important role in determining the brittleness of a cementitious matrix. The use of larger aggregate sizes increases the fracture energy of the concrete. To account for the increase in fracture energy of the matrix as the fibre volume concentration increases, due to the increase in matrix spalling and propagation of cracks around or fibre ends, for Mode I fracture, \( c_2 \) is taken as:

\[
c_2 = \frac{30}{1 + 100 \rho_f} \quad \text{.... for mortar and concrete with } a_g \leq 10 \text{ mm} \tag{4.3}
\]

\[
c_2 = \frac{20}{1 + 100 \rho_f} \quad \text{.... for concrete with } a_g > 10 \text{ mm} \tag{4.4}
\]

where \( a_g \) is the maximum size of the aggregate particles.

For Mode II fracture, the frictional resistance due to sliding will also increase fracture energy of the mortar component and \( c_2 \) is taken as:

\[
c_2 = \frac{4}{1 + 100 \rho_f} \tag{4.5}
\]

A total of 36 uniaxial tension (Mode I) tests undertaken by eleven independent investigators and eight direct shear (Mode II) tests carried out by Lee and Foster (2006) were compared and the results are presented below. The material data for all tests analysed is given in Appendix A.

### 4.2 PETERSSON (1980)

Petersson (1980) carried out a series of uniaxial tension tests for cement mortars reinforced with indented and straight steel fibres containing fibre volume concentrations of 0.25%, 0.5% and 1.0%.

The UVEM is compared against his experimental data for straight fibre series; the straight steel fibres used were 30 mm long by 0.3 mm in diameter. The specimen details and dimensions are shown in Figure 4.1. The specimen measured was 200 mm high and measured 20 mm by
50 mm a the critical section, which includes two 15 mm deep notches. For this geometry, the fibre boundary factor is calculated from Equation 3.35 as $K_b = 1.10$. The tensile strength of the unreinforced cement mortar matrix was determined experimentally as $f_{ct} = 3.0$ MPa but the compressive strength of the mortar was not reported. Hence, $\tau_{b,0}$ is taken as $0.9 f_{ct} = 2.7$ MPa (from Equation 4.2 and Table 4.1). Equation 3.2 is used to compute the stress versus crack opening displacement (COD) relationship for of the unreinforced mortar with $c_1 = 1$ and $c_2$ as per Equation 4.3.

The resulting stresses versus COD are plotted in Figure 4.2 and it is seen that the UVEM compares well against the experimental data.

### 4.3 LIM ET AL. (1987)

Lim et al. (1987) conducted eight series of uniaxial tension strength tests for end hooked and straight steel fibres in various volumetric ratios using conventional strength concrete with maximum aggregate size of 10 mm. Their specimen dimension and test arrangement are given in Figures 4.3. The specimen had a cross section of 70 mm by 100 mm and the fibre concentration ratios investigated were 0.5%, 1.0% and 1.5% by volume. Two types of steel fibres were used, end-hooked and straight steel fibres.

The straight steel fibres used were either 30 mm or 50 mm long, 0.57 mm in diameter and had a tensile strength of 345 MPa. The concrete mix comprised of cement, sand and coarse aggregate with ratios of 1 : 1.8 : 2.8 : 0.5. The coarse aggregate was local crushed granite of 10 mm. The factor $K_b$ is calculated as 1.07 for the 30 mm long fibres and 1.12 for the 50 mm long fibres (Equation 3.35). In the specimens using straight steel fibres, the tensile strength of the concrete matrix was measured as $f_{ct} = 2.19$ MPa but the compressive strength of the mortar was not reported by Lim et al. (1987). Therefore, $\tau_{b,0}$ is calculated using Equation 4.2 and Table 4.1, as $1.2 f_{ct} = 2.63$ MPa. In addition, Lim et al. undertook single fibre pullout tests to complement their tests. Thus, the actual values for $\tau_{b,0}$ are available and this data also used to obtain the stress versus COD response. The resulting UVEM stresses versus COD for straight steel fibres tests are plotted in Figures 4.4 to 4.7 and compare with the test results of Lim et al. (1987).
Figure 4.1: Uniaxial tensile test set-up of Petersson (1980).

Figure 4.2: Comparison of UVEM with uniaxial tension tests for cement mortar using 30 mm long by 0.3 mm diameter straight fibres by Petersson (1980).
Figure 4.3: Uniaxial tensile test and specimen dimension (in mm) of Lim et al. (1987).

![Uniaxial tensile test diagram]

**Figure 4.4:** Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 50 mm long by 0.57 mm diameter straight fibres with $\rho_f = 1.0\%$

- **Experimental Data**
- **UVEM ($\tau_{b,0,\text{calc}}$)**
- **UVEM ($\tau_{b,0,\text{measured}}$)**

**Straight Fibres**

- $l_f = 50\text{ mm}$
- $d_f = 0.57\text{ mm}$
- $\sigma_p = 340\text{ MPa}$
- $\rho_f = 1.0\%$

**Concrete**

- $f_{ct} = 2.19\text{ MPa}$
- $\tau_{b,0,\text{measured}} = 1.73\text{ MPa}$
- $\tau_{b,0,\text{calc}} = 2.63\text{ MPa}$
- $c_1 = 1$
- $K_b = 1.12$
- $c_2 = 15$
- $\beta = 10^\circ$
In their end hooked fibre tests, the fibres were 30 mm or 50 mm long, 0.5 mm diameter and had a tensile strength of 1130 MPa. By Equation 3.13, for the 50 mm long end hooked steel fibres used by the Lim et al. (1987), the critical embedded length is 48.2 mm, less than the fibre length and thus Equation 3.47 is used to calculate the global orientation factor, $K_f$. The $K_b$ for the 30 mm long fibres was determined by Equation 3.35 to be 1.07 and for the 50 mm long fibres is 1.12. The tensile strength of the concrete matrix was 2.66 MPa and the $\tau_{b,0}$ is estimated using Equation 4.2 and Table 4.1 as 6.38 MPa. The actual measured values for $\tau_{b,0}$ are also used to obtain the stress versus COD response. The results of the UVEM are compared with Lim et al. (1987)'s data in Figures 4.8 to 4.11. It is seen that UVEM compares relatively well against the experimental data.
Figure 4.6: Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 30 mm long by 0.57 mm diameter straight fibres with $\rho_f = 1.0\%$

Figure 4.7: Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 30 mm long by 0.57 mm diameter straight fibres with $\rho_f = 1.5\%$. 
Figure 4.8: Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 50 mm long by 0.5 mm diameter end hooked fibres with $\rho_f = 1.0\%$.

Figure 4.9: Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 30 mm long by 0.5 mm diameter end hooked fibres with $\rho_f = 0.5\%$. 
Figure 4.10: Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 30 mm long by 0.5 mm diameter end hooked fibres with $\rho_f = 1.0\%$.

Figure 4.11: Comparison of UVEM model with Mode I test data of Lim et al. (1987) for 30 mm long by 0.5 mm diameter end hooked fibres with $\rho_f = 1.5\%$. 
4.4 BEHLOUL (1995)

Behloul (1995) undertook a “dog bone” specimen uniaxial tension test on a 200 MPa ultra high performance reactive powder concrete (RPC). The RPC was reinforced by 12 mm long by 0.2 mm diameter straight steel fibres and fibre volumetric ratio was 2.6%. Figure 4.12 shows the specimen dimensions.

The factor $K_b$ for the Behloul specimen is calculated using Equation 3.35 as 1.04. The value of $\tau_{b,0}$ is estimated from the concrete compressive strength using Equation 4.1 and Table 4.1 and is given as 8.49 MPa. The results of UVEHM is compared in Figure 4.13 for the tensile stress versus COD. It is seen that the UVEHM gives a good correlation with the experimental data for both the peak stress and the descending curve.

![Figure 4.12: Tensile specimens of Behloul (1995).](image)

![Figure 4.13: Comparison of UVEHM model with experimental data of Behloul (1995) using 12 mm long by 0.2 mm diameter straight steel fibre reinforced RPC with $\rho_f=2.6\%$.](image)
4.5 NOGHABAI (2000)

Noghabai (2000) undertook a series of uniaxial tension tests containing 1% of straight steel fibres by volume in a high strength concrete matrix with a maximum aggregate size of 16 mm. The uniaxial tensile specimen dimensions are shown in Figure 4.14. The straight steel fibres used were 6 mm long and 0.15 mm diameter with ultimate tensile strength of 2600 MPa. The fibre boundary factor is calculated using Equation 3.35 and is $K_b = 1.03$. The unreinforced matrix compressive strengths were reported as $f_{cm} = 80$ MPa and 69.6 MPa for specimen denoted as HSC$^I_{S60/0.15}$ and HSC$^{III}_{S60/0.15}$, respectively. The measured tensile strengths for HSC$^I_{S60/0.15}$ and HSC$^{III}_{S60/0.15}$ were 4.15 MPa. The bond shear strengths of the fibre-matrix interfaces, calculated based on Equation 4.1 and Table 4.1, are 3.58 MPa and 3.34 MPa for HSC$^I_{S60/0.15}$ and HSC$^{III}_{S60/0.15}$, respectively. The results of the UVEM are compared in Figures 4.15 and 4.16. The figures show that the numerical results obtained from the UVEM compare well with Noghabai experimental data.

Figure 4.14: Uniaxial tensile specimen dimension of Noghabai (2000).
Figure 4.15: Comparison of UVEM model with the test data of Noghabai (2000) for 6 mm long by 0.15 mm diameter straight fibres with $\rho_f = 1.0\%$ from HSC$^{\text{I}}_{\text{S6/0.15}}$ matrix.

Figure 4.16: Comparison of UVEM model with the test data of Noghabai (2000) for 6 mm long by 0.15 mm diameter straight fibres with $\rho_f = 1.0\%$ from HSC$^{\text{III}}_{\text{S6/0.15}}$ matrix.
4.6 PLIZZARI ET AL. (2000)

Plizzari et al. (2000) investigated the post peak behaviour of fibre reinforced normal and strength concretes under cyclic tensile loads. In their investigation, a static test was undertaken for steel fibre reinforced concretes with a 10 mm maximum aggregate size. Details of the specimen are shown in Figure 4.17.

The steel fibres used by Plizzari et al. were end hooked, cold drawn, 30 mm long and had a diameter of 0.5 mm and a tensile strength of 1100 MPa. The volume fraction of the steel fibres was 0.38%. By Equation 3.35, the fibre boundary factor is $K_b = 1.07$. The measured compressive strength for the unreinforced concrete was 63.3 MPa and the measured tensile strength was 4.01 MPa. The bond shear strength of the fibre-matrix interface is estimated as per Equation 4.1 and Table 4.1, which gives $\tau_{b,0} = 6.36$ MPa. The resulting stresses versus crack opening displacement are presented in Figure 4.18 and it is observed that the UVE is strain.4.18 and it is observed that the UVEM compare well against the experimental data for the post peak cracking strength provided by the fibres.

Figure 4.17: Uniaxial tensile specimen dimension of Plizzari et al. (2000).
Figure 4.18: Comparison of UVEM model with the test data of Plizzari et al. (2000) for 30 mm long by 0.5 mm diameter end hooked fibres with $\rho_f = 0.38\%$ for normal strength concrete matrix.

4.7 GROTH (2000)

Groth (2000) tested four series of uniaxial tension tests on fibre reinforced cement mortars with a fibre volume concentration of 0.7%. The uniaxial tension tests were performed on notched cylindrical specimens, under closed looped displacement control, with the dimension as given in Figure 4.19. Dramix® straight fibres with 20 mm long and 0.13 mm diameter were used. The fibres had a tensile strength of 1000 MPa. The factor $K_b$ is determined as 1.04. (Equation 3.35).

For the Groth tests, four types of cement based matrices were used with varying quantities of ordinary Portland cement (OPC), silica fume, quartz sand and blast furnace slag. The four mixes were denoted as OPC, 50% of OPC and 50% of blast furnace slag (S50), 50% of OPC mixed with 50% of quartz sand (Q50), and 95% OPC with 5% of silica fume (EMC500). Table 4.2 shows the measured compressive ($f_{cm}$) and tensile ($f_{ct}$) strengths for the plain matrices. The fibre bond strengths, $\tau_{b,0\text{(calc)}}$, are calculated using Equation 4.1 and Table 4.1. In addition, the measured fibre bond strengths, $\tau_{b,0\text{(measured)}}$ by Groth (2000) using single fibre pullout tests were also presented and the actual bond strength values are also used to obtain the
stress versus COD response. The results of the UVEM are compared in Figures 4.20 to 4.23 and, again, it is seen that UVEM compared well against the experimental data.

Table 4.2: Material properties of Groth (2000).

<table>
<thead>
<tr>
<th>Matrix</th>
<th>$f_{cm}$ (MPa)$^\dagger$</th>
<th>$f_{ct}$ (MPa)</th>
<th>$\tau_{b.0}$ (calc) (MPa)</th>
<th>$\tau_{b.0}$ (measured) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPC</td>
<td>45.3</td>
<td>2.6</td>
<td>2.02</td>
<td>1.5</td>
</tr>
<tr>
<td>S50</td>
<td>49.4</td>
<td>3.2</td>
<td>2.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Q50</td>
<td>28.8</td>
<td>3.2</td>
<td>1.61</td>
<td>1.2</td>
</tr>
<tr>
<td>EMC500</td>
<td>51.9</td>
<td>2.4</td>
<td>2.16</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Note: $^\dagger$ Taken as 0.8 of the cube compressive strength.

Figure 4.19: Uniaxial tensile specimen dimension of Groth (2000).
Figure 4.20: Comparison of UVEM with uniaxial tension test from Groth (2000): OPC series.

Figure 4.21: Comparison of UVEM with uniaxial tension test from Groth (2000): S50 series.
Figure 4.22: Comparison of UVEM with uniaxial tension test from Groth (2000): Q50 series.

Figure 4.23: Comparison of UVEM with uniaxial tension test from Groth (2000): EMC500 series.
4.8 DENARIÉ ET AL. (2003)

Denarié et al. (2003) reported the uniaxial tension strength of an RPC mix containing 2% of straight steel fibres by volume. The uniaxial tension tests were performed on notched prism specimens, with the dimensions given in Figure 4.24. Straight steel fibres of 13 mm long and 0.15 mm was used, giving a fibre boundary influence factor by Equation 3.35 of $K_b = 1.04$. The cylinder compressive strength of the RPC was measured as 171 MPa and, thus, the bond shear strength, $\tau_{b,0}$, of the fibre-matrix interface is estimated using Equation 4.1 and Table 4.1, as 7.85 MPa. The tensile strength of the matrix is taken as 7.5 MPa. The result of the UVEM is compared against the experimental data in Figure 4.25 and shows that the UVEM compares well with the Denarié et al. (2003)'s data for the stress versus COD.

![Diagram](image-url)

**Figure 4.24:** Uniaxial tensile test and specimen dimension of Denarié et al. (2003).
Figure 4.25: Comparison of UVEM with uniaxial tension test from Denarié et al. (2003).

4.9 BARRAGÁN ET AL. (2003)

Barragán et al. (2003) tested a series of five uniaxial tension strength specimens containing 0.45% of end hooked fibres. The fibres were 60 mm long by 0.75 mm in diameter and had a tensile strength of 1000 MPa. The test specimen dimension and testing arrangements are given in Figure 4.26. The boundary factor $K_b = 1.08$ as per Equation 3.35. The concrete used has a maximum aggregate size of 12 mm, with aggregate to cement ratio of 5.3 and water to cement ratio of 0.57. The compressive strength of the concrete was 41 MPa and $\tau_{b,0}$ is taken as 5.12 MPa (Equation 4.1 and Table 4.1). The results of the UVEM are compared with the test data in Figure 4.27 for the tensile strength versus COD. It is seen that the UVEM compares well against the average of Barragán et al. data.
Figure 4.26: Uniaxial tensile test and specimen dimension of Barragán et al. (2003).

![Diagram](image)

Figure 4.27: Comparison of UVEM model with the test data of Barragán et al (2003) for normal strength concrete reinforced with 0.45% by volume of 60 mm long by 0.75 mm diameter end hooked fibres.
4.10 Sorelli et al. (2005)

Sorelli et al. (2005) tested a series of fibre reinforced concrete specimens with 0.38 percent by volume of steel fibres. Straight micro and macro steel fibres were used in their investigation. The macro fibres had a length of 30 mm and a diameter of 0.6 mm, whereas the micro fibres had a length of 12 mm and a diameter of 0.18 mm. The tensile strengths were 1100 MPa for the macro fibres and 1800 MPa for the micro fibres. The specimen details and dimension are presented in Figure 4.28. The specimen dimension was 100 mm by 40 mm by 200 mm high with two 15 mm deep notches. Therefore, the fibre boundary factor, $K_b$, is determined using Equation 3.35 and is 1.04 for the micro fibres. For the macro fibres, $K_b$ is 1.12. Three different types of fibre combinations were tested, namely 0.38% by volume of micro fibres, 0.38% by volume of macro fibres, and a hybrid of 0.38% by volume of micro fibres and macro fibres. The concrete has a maximum aggregate size of 15 mm. The compressive strength and tensile strength of the unreinforced cement mortar matrix were determined experimentally as 22.6 MPa and 2.85 MPa, respectively. The initial bond strength $\tau_{b,0} = 1.9$ MPa, estimated from the concrete compressive strength using Equation 4.1 and Table 4.1. The results of the UVEM are compared in Figures 4.29 to 4.31 for the tensile stress versus COD. It is seen that the UVEM fits within the bounds of the experimental data, albeit at the lower (conservative) end of the data range.

![Figure 4.28: Tensile test specimen of Sorelli et al. (2005).](image-url)
Figure 4.29: Comparison of UVEM with the test data of Sorelli et al. (2005) for 12 mm long by 0.18 mm diameter micro straight fibres with $\rho_f = 0.38\%$.

Figure 4.30: Comparison of UVEM with the test data of Sorelli et al. (2005) for 30 mm long by 0.6 mm diameter macro straight fibres with $\rho_f = 0.38\%$. 

---

**Micro-Straight Fibres**

- $l_f = 12\ mm$
- $d_f = 0.18\ mm$
- $\rho_f = 0.38\%$
- $\sigma_{ph} = 1800\ MPa$

**Concrete**

- $f_{cm} = 22.6\ MPa$
- $f_{ct} = 2.85\ MPa$
- $\tau_{s0} = 1.9\ MPa$

- $c_i = 1$
- $K_p = 1.04$
- $c_2 = 14.5$
- $\beta = 3.8^\circ$

---

**Macro-Straight Fibres**

- $l_f = 30\ mm$
- $d_f = 0.6\ mm$
- $\rho_f = 0.38\%$
- $\sigma_{ph} = 1100\ MPa$

**Concrete**

- $f_{cm} = 22.6\ MPa$
- $f_{ct} = 2.85\ MPa$
- $\tau_{s0} = 1.9\ MPa$

- $c_i = 1$
- $K_p = 1.12$
- $c_2 = 14.5$
- $\beta = 3.8^\circ$
Figure 4.31: Comparison of UVEM with the test data of Sorelli et al. (2005) for hybrid micro and macro fibre combination with total $\rho_f = 0.38\%$.

4.11 SUSETYO (2010)

Susetyo (2010) investigated the effectiveness of hooked-end steel fibres to control cracks. Eight series of uniaxial tension tests were carried out for steel fibre reinforced concrete using the test configurations as per Figure 4.32.

In his experiment, the fibre concentration ratios investigated were 0.5%, 1.0% and 1.5% by volume and two grades of concrete strength were used: normal strength and high strength. Dramix® RC-80/50-BN, RC-65/60-BN and RC-80/30-BP end hooked fibres were used. The Dramix® RC-80/50-BN fibres were 50 mm long by 0.62 mm diameter and had a tensile strength of 1050 MPa. On the other hand, Dramix® RC-65/60-BN fibres were 35 mm long by 0.55 mm diameter and had a tensile strength of 1100 MPa while Dramix® RC-80/30-BP fibres were 30 mm long by 0.38 mm diameter and are reported tensile strength of 2300 MPa. By Equation 3.35, the fibre boundary factors, $K_b$, are calculated as 1.12 for Dramix® RC-80/50-BN fibres, 1.08 for Dramix® RC-65/35-BN fibres and 1.07 for Dramix® RC-80/30-BP fibres.

The maximum aggregate size used was 10 mm in the concrete mixtures. The unreinforced matrix compressive strength were reported as $f_{cm} = 65.7$ MPa and 90.5 MPa for specimen pre-
fixed with C1 and C2, respectively. In addition, the measured tensile strengths for C1 and C2 were 4.07 MPa and 4.13 MPa, respectively. The fibre bond shear strengths, calculated based on Equation 4.1 and Table 4.1, were 6.48 MPa and 7.61 MPa for C1 and C2, respectively. For C1F1V1, C1F1V2, C1F1V3, C2F1V3 and C2F3V3 series, by Equation 4.16, the fibre critical embedded lengths are lesser than the fibre length and, thus, Equation 4.53 is used to calculated the global orientation factors, $K_f$.

The resulting UVEM stresses versus COD are plotted in Figures 4.33 to 4.40 and compared with that of Susetyo (2010). The agreement is varied, the model predicts the test results sufficiently well for the important first two millimeters, noting that the test data scatter for the experiment is significant. Beyond the first two millimeters of COD, the model generally over predicts the results for the Susetyo tests and the failure mechanics of these tests at the longer displacement range require closer examination.

Figure 4.32: Uniaxial tension test configurations of Susetyo (2010).
Figure 4.33: Comparison of UVEM model with Susetyo (2010)’s C1F1V1 test series.

Figure 4.34: Comparison of UVEM model with Susetyo (2010)’s C1F1V2 test series.
Figure 4.35: Comparison of UVEM model with Susetyo (2010)'s C1F1V3 test series.

Figure 4.36: Comparison of UVEM model with Susetyo (2010)'s C1F2V3 test series.
Figure 4.37: Comparison of UVEM model with Susetyo (2010)’s C1F3V3 test series.

Figure 4.38: Comparison of UVEM model with Susetyo (2010)’s C2F1V3 test series.
**End Hooked Fibres**

- $l_f = 30$ mm
- $d_f = 0.38$ mm
- $\rho_f = 1.5\%$
- $\sigma_{fu} = 2300$ MPa

**Concrete**

- $f_c = 90.5$ MPa
- $f_{ct} = 4.13$ MPa
- $\tau_{b0} = 7.61$ MPa
- $c_1 = 1$
- $K_n = 1.07$
- $c_2 = 12$
- $\beta = 30^\circ$

Figure 4.39: Comparison of UVEM with Susetyo (2010)’s C2F2V3 test series.

---

**End Hooked Fibres**

- $l_f = 35$ mm
- $d_f = 0.55$ mm
- $\rho_f = 1.5\%$
- $\sigma_{nu} = 1100$ MPa

**Concrete**

- $f_c = 90.5$ MPa
- $f_{ct} = 4.13$ MPa
- $\tau_{b0} = 7.61$ MPa
- $c_1 = 1$
- $K_n = 1.08$
- $c_2 = 12$
- $\beta = 30^\circ$

Figure 4.40: Comparison of UVEM with Susetyo (2010)’s C2F3V3 test series.
4.12 HTUT (2010)

Htut (2010) investigated the uniaxial tension behaviour of cement mortar reinforced with 0.5%, 1.0%, 1.5% and 2.0% by volume of end hooked fibres. The fibres were 0.3 mm in diameter, had a length of 25 mm and had a minimum tensile strength of 2300 MPa. The test was conducted on a 30 mm thick “dog bone” specimen. Details of the specimen and the testing arrangement are similar to that of uniaxial tension test conducted in this study, discussed in Section 4.4.

By Equation 3.35, the fibre boundary influence factor \( K_b = 1.14 \). The tensile strength of the plain matrix was measured as 2.7 MPa and the bond shear strength of the fibre-matrix interface was determined to be 8.4 MPa. The limiting maximum fibre content for Htut (2010) experiments was found to be \( \rho_{f,\text{limit}} = 0.5\% \), as discussed in Section 4.8. The results of the UVEM are compared against the experimental data in Figures 4.41 to 4.44. While the model compares sufficiently well with the test data, modelling of tests that are limited by the strength of the matrix remains problematic and further study is needed.

![Graph showing comparison of UVEM model with Mode I test data of Htut (2010)](image)

Figure 4.41: Comparison of UVEM model with Mode I test data of Htut (2010) with \( \rho_f = 0.5\% \).
Figure 4.42: Comparison of UVEM model with Mode I test data of Htut (2010) with \( \rho_f = 1.0\% (\rho_{f, limit} = 0.5\%). \)

Figure 4.43: Comparison of UVEM model with Mode I test data of Htut (2010) with \( \rho_f = 1.5\% (\rho_{f, limit} = 0.5\%). \)
Figure 4.44: Comparison of UVEM model with Mode I test data of Htut (2010) 
with $\rho_f = 2.0\% (\rho_{f,\text{limit}} = 0.5\%)$.


The Mode II fracture tests of Lee and Foster (2006), with 35 mm long by 0.55 mm diameter end-hooked fibres and 13 mm long by 0.2 mm diameter straight fibres and with fibre volumetric ratios of $\rho_f = 0.5\%, 1.0\%, 1.5\%$ and $2\%$, are used to verify the performance of the model. Figure 4.45 presents the detail test set-up of Lee and Foster (2006)'s experiments. By Equation 3.35, the fibre boundary factor is calculated as $K_b = 1.06$ for the end hooked fibres and for the straight fibres is 1.03. The tensile strength of the cement mortar used was found to be 4.4 MPa and the $\tau_{b,0}$ were found to be 4 MPa and 8.8 MPa for the straight fibre and the end hooked fibre tests, respectively. In the verification process, Equation 3.2 is used to determine the unreinforced mortar matrix stress versus COD relationship. Coefficient $c_1$ is taken as $1 + 72\rho_f$ and $c_2$ is determined using Equation 4.5.

The results of the Lee and Foster end hooked fibre test series are plotted in Figure 4.46. It is seen that the UVEM model compares well against the experimental results for the residual strength of the SFRC, up to CMODs exceeding 10 mm, but significantly under estimates the
early matrix component. This is likely due to friction between the surfaces not accounted for in the model. The results for the straight fibre series are plotted in Figure 4.47 and it is seen that the UVEK provides conservative predictions for these tests.

Figure 4.45: Direct shear test configurations of Lee and Foster (2006).

### 4.14 CONCLUSIONS

The proposed Unified Variable Engagement Model (UVEK) has been validated against a wide range of experimental data, including uniaxial tension (Mode I) tests and direct shear (Mode II) tests on fibre reinforced concretes and mortars by twelve researchers. In the verification studies, the influence of boundary surfaces were considered via the fibre boundary influence factor, $K_b$. Overall the model showed a good correlation with the experimental results. While the model is developed to include mixed mode fracture, as yet no test data is available other than for fracture Modes I and II. Thus, further data is needed to fully validate the model for mixed mode fracture.
--- Experimental Data
--- UVEM

**End-Hooked Fibres**

\[ l_f = 35 \text{ mm} \quad d_f = 0.55 \text{ mm} \quad \sigma_{pu} = 1345 \text{ MPa} \]

**Cement Mortar**

\[ f_{ct} = 4.4 \text{ MPa} \quad \tau_0 = 8.8 \text{ MPa} \]

\[ K_h = 1.06 \quad c_i = 1 + 72\rho_f \]

---

**Straight Fibres**

\[ l_f = 13 \text{ mm} \quad d_f = 0.2 \text{ mm} \quad \sigma_{pu} = 2200 \text{ MPa} \]

**Cement Mortar**

\[ f_{ct} = 4.4 \text{ MPa} \quad \tau_0 = 4 \text{ MPa} \]

\[ K_s = 1 \quad K_{sh} = 1 \quad K_h = 0.83 \]

\[ c_i = 1 + 72\rho_f \]

---

Figure 4.46: Comparison of UVEM model with the direct shear test data of Lee and Foster (2006) for end hooked fibre reinforced cement mortar.

---

Figure 4.47: Comparison of UVEM model with the direct shear test data of Lee and Foster (2006) for straight fibre reinforced cement mortar.
CHAPTER 5  CONCLUSIONS

Cementitious composites are quasi-brittle materials with low tensile strength and strain capacity. The use of fibres as a reinforcement in these materials is becoming more common and serves the purpose of both increasing the tensile strength and improving the post-cracking behaviour of the material with the fibres bridging the crack openings.

In 2003 Voo and Foster (2003, 2004) proposed a simple model for the tensile behaviour of steel fibre reinforced concrete. The model was based on integrating the various components crossing a fracture plane, viz the matrix component together with summing the individual fibre components for all fibres embedded on one side and pulling out from the other. To this end, a simple relationship was proposed for the fibres with different fibres at different angles engaging at different times. The model was termed the Variable Engagement Model or VEM.

In this study the VEM has been revised with consideration to the additional data and experimental observations that have occurred since the model was first conceived. The model has also been extended to include Mode II and Mixed Mode fracture, in addition to that of Mode I. The influence of near field boundaries is also included in the updated model.

The model has been validated against a wide range of data collected by a number of investigators with the data set including a range of conventional steel fibre reinforced concrete and mortars, as well high-performance reactive powder concrete. In all, data from thirty-six uniaxial tensile tests (Mode I) and eight direct shear fracture tests (Mode II) are examined that were tested by twelve researchers. Overall the model shows a good correlation with the test results with the post-cracking behaviour predicted by the model comparable with that of the experiments.
REFERENCES


This Appendix contains the material property data used Unified Variable Engagement Model developed in Chapter 3 and tested and evaluated in Chapter 4 of this report.
Table A.1: Material property data of Petersson (1980) and Lim et al. (1987).

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**Measured Composite Properties**

| $\rho_f$ (%) | 0.5 | 1.0 | 1.5 | 1.0 | 2.6 | 0.7 | 0.7 | 0.7 | 0.7 |
| $f_{ct}$ (MPa) | 2.78 | 3.04 | 3.03 | 2.84 | 11 | 3.3 | 4.2 | 3.7 | 3.3 |
| $f_{cm}$ (MPa) | - | - | - | - | 200 | 44.1$^+$ | - | - | - |
| $\tau_{b,0}$ (MPa) | 6.87 | 6.72 | 7.10 | 5.85 | - | 1.5 | 1.5 | 1.2 | 2.0 |

$^+$ Taken as 0.8 of the cube compressive strength
Table A.3: Material property data of Noghabi (2000), Plizzari et al. (2000), Denarié et al. (2003) and Sorelli et al. (2005)

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Table A.4: Material property data of Barragán et al. (2003) and Suseyto (2010).

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**Measured Composite Properties**

| $\rho_f$ (%) | 0.45 | 0.5  | 1.0  | 1.5  | 1.5  | 1.5  | 1.5  | 1.5  |
| $f_{ct}$ (MPa) | 2.6  | 3.75 | 3.46 | 4.34 | 3.92 | 4.76 | 4.68 | 4.32 |
| $f_{cm}$ (MPa) | -    | 51.4 | 53.4 | 49.7 | 45.7 | 78.8 | 76.5 | 62.0 |
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Table A.5: Material property data of Htut (2010).

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<td>35</td>
<td>35</td>
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</tr>
<tr>
<td>$d_f$ (mm)</td>
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<td>0.55</td>
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<td>0.55</td>
</tr>
<tr>
<td>$E_f$ (GPa)</td>
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<td>-</td>
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<tr>
<td>$\sigma_{fu}$ (MPa)</td>
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**Measured Matrix Properties**

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<tr>
<td>$f_{ct}$ (MPa)</td>
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<tr>
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**Measured Composite Properties**

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<td>$f_{ct}$ (MPa)</td>
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<td>49.5</td>
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Table A.6: Material property data of Lee and Foster (2006).

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<th>Designation</th>
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Steel Fibre Properties

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<th>Straight</th>
<th>Straight</th>
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<th>End hooked</th>
<th>End hooked</th>
<th>End hooked</th>
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<td>13</td>
<td>13</td>
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<td>35</td>
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<td>35</td>
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<tr>
<td>$d_f$ (mm)</td>
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<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
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<tr>
<td>$E_f$ (GPa)</td>
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Measured Matrix Properties

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<td>Normal</td>
<td>Normal</td>
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<td>28</td>
<td>28</td>
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<td>52</td>
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<td>49.5</td>
<td>49.5</td>
</tr>
</tbody>
</table>

Measured Composite Properties

| $\rho_f$ (%) | 0.5 | 1.0 | 1.5 | 2.0 | 0.5 | 1.0 | 1.5 | 2.0 |
| $f_{ct}$ (MPa) | -   | -   | -   | -   | -   | -   | -   | -   |
| $f_{cm}$ (MPa) | 52  | 49.5| 49  | 48.5| 49.5| 49.5| 44  | 52  |
| $\tau_{60}$ (MPa) | 3.96| 3.96| 3.96| 3.96| 8.8 | 8.8 | 8.8 | 8.8 |