TIME-DEPENDENT STRESSES AND DEFORMATIONS IN COMPOSITE BEAMS

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7. Abstract

A theoretical treatment is presented of the analysis of a composite  
steel-concrete cross-section in sagging bending. The analysis uses the  
Age-Adjusted Effective Modulus Method to model the creep and shrinkage  
behaviour of the concrete. Strains predicted from the model are compared  
with those obtained from test results, and the agreement is shown to be good.  
The theoretical model is used to calculate the influence of the material  
properties and slab reinforcement on the stresses in the concrete slab and  
in the steel joist.

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TIME-DEPENDENT STRESSES AND STRAINS
IN COMPOSITE BEAMS

by

M.A. Bradford and R.I. Gilbert

INTRODUCTION

Steel-concrete composite beams are used widely in office building construction where the compressive strength of the concrete floor slab complements the tensile strength of the steel joist. Two-way composite action is often achieved by the use of profiled steel sheeting which acts as both tensile reinforcement and permanent formwork for the concrete slab spanning between the steel joists.

The limit states design procedure requires a design that meets both strength and serviceability criteria. In sagging bending, the strength limit state usually corresponds to plastification of the cross-section, while strength in hogging bending near an internal support or frame connection is often governed by buckling of the joist. On the other hand, the serviceability criterion most often met in practice is the limiting of excessive deflections at service loads. This paper is concerned with behaviour at service loads.

For the serviceability limit state, the time-varying properties of the concrete must be considered in calculating the deflections. This is often done in codes of practice by adjusting the short-term elastic modulus of the concrete and using routine modular ratio theory. However, this does not take proper account of concrete shrinkage, which can have a major bearing on the time-dependent deflections of a composite beam under a sustained load.

Several predictive models for the behaviour of a structure subject to concrete creep and shrinkage are available, and these are presented in recent state of the art texts [1,2,3]. Although these have been applied extensively to reinforced and prestressed concrete structures, the application of these models to composite steel-concrete structures, although straightforward, has received less attention. With regard to creep and shrinkage, the time-dependent behaviour of composite columns has been studied.
by Behan and O'Connor [4], Bridge [5], Bradford and Gilbert [6], Bradford [7], and others, while the behaviour of composite beams at service loads has also been considered by Roll [8], Gilbert [9], Bradford and Gilbert [10,11,12], Johnson [13] and Bradford [14].

Until recently, very little testing of composite beams under service loads had been undertaken, with the 1971 work of Roll [8], although of very limited applicability, being most often quoted. Experimental work was also undertaken at the University of Warwick by Johnson [13]. In order to supplement the limited available experimental data, the authors undertook a series of tests on four full-scale simply supported composite tee-beams at the University of New South Wales [11]. The primary aim of these tests was to validate the simple design proposal for deflections presented in Refs. 12 and 14. It was shown in Ref. 11 that deflections calculated according to this design proposal were in close agreement with the measured deflections in the test beams. The strains measured in the steel and concrete were not given in Ref. 11, and these are reported herein, together with a comparison between the measured and the computed deformations.

The theoretical model presented in Ref. 9 accounted for nonlinear behaviour of the steel at service loads resulting from residual stresses. However, it was shown in this study that at typical service loads, the effects of nonlinearity of the steel are negligible. Because of this, a model for linear behaviour, originally proposed by the second author in Ref. 3, is summarised in the present paper. The steel and concrete strains are validated against those measured in the tests [11], and the stresses and deformations are calculated using an age-adjusted effective modulus [15] to account for the time-dependent creep deformations in the concrete. Finally, the influences of the creep coefficient, shrinkage strain and slab reinforcement ratio on the stresses in the concrete and in the steel joist are demonstrated.

THEORETICAL MODEL

A method for the time-dependent analysis of a composite steel-concrete section was given in Ref. 3, and a summary which lends itself to hand calculation or simple programming is given in this section.

Figure 1 shows the composite cross-section and initial strain distribution. It is
assumed that slip is negligible, so that the strains are linear over the full depth of the cross-section. It is further assumed that the concrete is uncracked. Compressive stresses are positive, and a positive bending moment on the section results in tension in the bottom fibres of the steel joist.

Consider firstly the transformed cross-section shown in Fig. 1, where the modular ratio \( n \) is given by \( E_s/E_c \), in which \( E_s \) is the elastic modulus of the steel and \( E_c \) is the short-term elastic modulus of the concrete slab. The area of the transformed section \( A \), and the first and second moments of area of the transformed section about the top fibre, \( B \) and \( I \) respectively, are

\[
A = b D_c + (n-1) A_{sr} + n A_{ss} \tag{1}
\]

\[
B = b D_c^2/2 + (n-1) A_{sr} d_{sr} + n A_{ss} d_{ss} \tag{2}
\]

\[
I = b D_c^3/3 + (n-1) A_{sr} d_{sr}^2 + n(I_{ss} + A_{ss} d_{ss}^2) \tag{3}
\]

where \( A_{ss} \) is the cross-sectional area of the steel joist, \( I_{ss} \) is the second moment of area of the steel joist about its own centroidal axis, and the geometrical properties \( b, D_c, d_{sr} \) and \( d_{ss} \) are given in Fig. 1.

If \( \varepsilon_{oi} \) is the initial top fibre strain in the section and \( \rho_i \) is the curvature, the strain \( \varepsilon_i \) at any depth \( y \) below the top reference fibre may be written as

\[
\varepsilon_i = \varepsilon_{oi} - y \rho_i \tag{4}
\]

The short-term axial force \( N_i \) on the section (which is zero for the case of pure bending considered here) may be obtained by integrating the short-term stresses shown in Fig. 1 as

\[
N_i = \int E_c \varepsilon_i dA \tag{5}
\]

so that, from Eq. 4,

\[
N_i = E_c \varepsilon_{oi} A - E_c \rho_i B \tag{6}
\]
where A and B are given in Eqs. 1 and 2 respectively. The short-term moment $M_i$ taken about the top reference surface may be obtained by integrating the first moment of the stress block about the top reference surface. Hence

$$M_i = - \int E_c \varepsilon_i y \, dA$$  \hspace{1cm} (7)

so that, from Eq. 4,

$$M_i = -E_c \varepsilon_{oi} B + E_c \rho_i I$$  \hspace{1cm} (8)

where $I$ is given by Eq. 3.

Solving Eqs. 6 and 8 simultaneously for $\varepsilon_{oi}$ and $\rho_i$ produces

$$\varepsilon_{oi} = \frac{B M_i + I N_i}{E_c (AI - B^2)}$$  \hspace{1cm} (9)

and

$$\rho_i = \frac{A M_i + B N_i}{E_c (AI - B^2)}$$  \hspace{1cm} (10)

Of course, the initial stress in the concrete slab $\sigma_{ci}$, in the steel joist $\sigma_{si}$ and in the steel reinforcement $\sigma_{sri}$ are simply obtained from

$$\sigma_{ci} = E_c (\varepsilon_{oi} - y \rho_i) \quad y < D_c$$  \hspace{1cm} (11)

$$\sigma_{si} = E_s (\varepsilon_{oi} - y \rho_i) \quad y \geq D_c$$  \hspace{1cm} (12)

$$\sigma_{sri} = E_s (\varepsilon_{oi} - d_{sr} \rho_i)$$  \hspace{1cm} (13)

For the time analysis, the relaxation procedure outlined in Ref. 3 is used. The restraining axial force and bending moment $- \Delta N$ and $- \Delta M$ respectively which are required to prevent the free development of creep and shrinkage in the concrete slab are [3]
\[-\Delta N = -\bar{E}_e \left\{ \phi (A_c \varepsilon_{o1} - B_c \rho_i) + \varepsilon_{sh} A_c \right\} \]  
\[-\Delta M = -\bar{E}_e \left\{ \phi (-B_c \varepsilon_{o1} + I_c \rho_i) - \varepsilon_{sh} B_c \right\} \]  

where $A_c$, $B_c$ and $I_c$ are the area, first moment of area and second moment of area of the concrete slab with reference to the top fibre, $\phi$ is the creep coefficient associated with the time interval under consideration, $\varepsilon_{sh}$ is the shrinkage strain during the same time period, and $\bar{E}_e$ is the age-adjusted effective modulus given by

\[
\bar{E}_e = \frac{E_e}{1 + \chi \phi}
\]

in which $\chi$ is the aging coefficient defined and discussed in Ref. 3.

By using the transformed section properties $\bar{A}_e$, $\bar{B}_e$ and $\bar{I}_e$ corresponding to $A$, $B$ and $I$ but using a modular ratio $\bar{n}$ of $E_s / \bar{E}_e$, the time-dependent change of top fibre strain $\Delta \varepsilon_o$ and curvature $\Delta \rho$ may be obtained, similarly to Eqs. 9 and 10, as [3]

\[
\Delta \varepsilon_o = \frac{\bar{B}_e \Delta M + \bar{I}_e \Delta N}{\bar{E}_e (\bar{A}_e \bar{I}_e - \bar{B}_e^2)} \]  
\[
\Delta \rho = \frac{\bar{A}_e \Delta M + \bar{B}_e \Delta N}{\bar{E}_e (\bar{A}_e \bar{I}_e - \bar{B}_e^2)} \]

The final long-term top fibre strain is $\varepsilon_{o1} + \Delta \varepsilon_o$ and the final time-dependent curvature is $\rho_i + \Delta \rho$.

Finally, addition of the stress loss that occurs during the relaxation procedure and the gain in stress that occurs when $\Delta N$ and $\Delta M$ are applied to the cross-section gives the change in stress $\Delta \sigma_c$ in the concrete slab at any point $y$ below the top fibre. Hence, from Ref. 3, the change in concrete stress is
\[ \Delta \sigma_c = -\bar{E}_c \left( \phi \( \varepsilon_{o1} - y \rho_i \) + \varepsilon_{sh} - (\Delta \varepsilon_o - y \Delta \rho) \right) \] (19)

while in the slab reinforcement, the stress change \( \Delta \sigma_{sr} \) due to creep and shrinkage is

\[ \Delta \sigma_{sr} = E_s (\Delta \varepsilon_o - d_{sr} \Delta \rho) \] (20)

and the stress change \( \Delta \sigma_{ss} \) in the steel joist, for \( y > D_c \), is

\[ \Delta \sigma_{ss} = E_s (\Delta \varepsilon_o - y \Delta \rho) \] (21)

Calculation of the strains and stresses, both immediate and time-dependent, may be achieved readily by hand calculation, provided the time-dependent properties \( \phi \), \( \varepsilon_{sh} \) and \( \chi \) are known. However, the method was automated herein by the writing of a short Pascal computer program. Values of the relevant concrete material properties may be obtained from the data given in the state of the art texts [1,2,3].

**EXPERIMENTAL VERIFICATION**

To validate the theoretical model, an experimental programme was commenced at the University of New South Wales. The details of long-term tests on four simply-supported composite beams are given in Ref. 11. Two of these beams had shear studs spaced at 200mm intervals in rows of two, and may be considered to have negligible slip at the steel-concrete interface. Of these two, beam B1 had a sustained load of 7.52 kN/m applied, while beam B2 was subjected to self-weight loading only. The geometry of the beams is shown in Fig. 2, and these two beams were selected as the theoretical model assumes negligible slip.

Measurements were made of shrinkage and creep strains on companion concrete specimens under the same ambient conditions as the test beams. The measured shrinkage strains \( \varepsilon_{sh} \) on three specimens are given in Fig. 3, while the average creep coefficient \( \phi(t,10) \) for loading at \( t = 10 \) days is given in Fig. 4. The value of the Young's modulus \( E_c \) of the concrete in Eqs. 9,10 and 16 was measured to be 25,210 MPa, and a value of the aging coefficient \( \chi \) of 0.85 was assumed [3].

Using the theoretical model of the previous section with the relevant material
properties $e_{sh}$ and $\phi$ from Figs. 3 and 4 respectively, the strains at midspan were calculated in (i) the top surface of the concrete slab; (ii) the soffit of the concrete slab; and (iii) the bottom fibre of the steel joist. The calculated strains are compared with the actual strains measured using Demec gauges in the laboratory.

Figure 5 shows the measured and calculated strains in the top surface of the slab. The measured strains were averaged from two sets of Demec targets placed near the beam centreline. The same comparison is given in Fig. 6 for the strains in the soffit of the slab. Here, Demec targets were placed close to each flange tip, and the average measured strain was taken. Finally, Fig. 7 shows the calculated and measured steel strains in the bottom fibre of the steel joist, with the measured strains being obtained from the average of those measured midway across each flange outstand. In all cases, the agreement between the measurements and theoretical predictions is satisfactory, with closer agreement being obtained for the beam B1 under an applied sustained load.

Good agreement between test and theory was also obtained in Ref. 11 for midspan deflections (the results of which are not presented herein), the latter being calculated by integrating the curvatures from the model of the previous section. It may thus be concluded that the transformed section analysis incorporating the age-adjusted effective modulus method and relaxation procedure is a reliable model for determining the strains and deformations in composite steel–concrete beams at service loads.

**APPLICATION OF THEORY**

The theoretical model was used to study the same composite section used in the experiments in order to obtain stresses in the concrete and steel. For this simply-supported 5.9m span, long-term behaviour was modelled by using two different values for the creep coefficient and shrinkage strain, namely $\phi = 3$ and $e_{sh} = 600 \ \mu e$ (representing final long-term values) and $\phi = 2$ and $e_{sh} = 400 \ \mu e$ (which represent values at age about 6 months). The sustained load was varied from $q = 1.5$ kPa, which represents self-weight, up to a maximum value of $q = 8$ kPa.

Figure 8 shows the instantaneous and time-dependent stress in the top of the slab at midspan for various values of the slab reinforcement ratio $p$. It can be seen that increasing the reinforcement ratio decreases the time-dependent compressive stress, and that the compressive stresses are gradually decreased with time. Under sustained
self-weight, the long-term stresses in the top of the slab may reach a tensile value of about 1 MPa.

The stresses in the soffit of the slab at midspan are shown in Fig. 9. The figure shows that the stresses are predominantly tensile, and increasing the amount of reinforcement in the slab increases the tensile stress in the concrete up to about 2 MPa for dead-weight loading. If the concrete is of low strength, this might lead to cracking of the slab for long-term loading. However, durability considerations would probably lead to the use of a concrete with compressive strength in excess of 25 MPa, so that shrinkage-induced cracking would be unlikely.

Figure 10 shows the tensile stresses in the bottom flange of the steel joist. These are well below yield even for high values of q, and are fairly insensitive to the slab reinforcement ratio. The tensile stresses increase with time as the slab is effectively unloaded by its time-dependent creep and shrinkage deformation. It may thus be concluded that steel stress variations due to creep and shrinkage, although relatively large, are not significant in design. It is the increase in deformation that is significant [11].

Finally, Figs. 11 and 12 show the top and bottom concrete slab stresses respectively under a sustained load q of 5 kPa. Again, the longer-term compressive stresses are lower than the shorter-term stresses (Fig. 11), and the long-term soffit tensile stresses can be reasonably significant for a large amount of slab reinforcement (Fig. 12). It would, however, be unlikely that a low strength concrete slab with p > 0.02 would be used.

CONCLUSIONS

The paper presents a theoretical treatment of the short-term and time-dependent behaviour of a steel-concrete composite cross-section. The method is linear, and differs from the more general nonlinear treatment presented by Bradford and Gilbert in Ref. 9. Time-dependent loading is modelled by use of the age-adjusted effective modulus method and a relaxation procedure. Both of these concepts are documented in detail elsewhere.
The equations for the initial and time–dependent top fibre strains and curvatures are relatively simple, and allow either hand calculation or a short computer program to be used to calculate the deformations and the stresses. Integration of the curvatures twice allows the load–deformation history to be derived.

A series of tests undertaken by the authors was reported briefly, and the results of these tests were used to validate the computed strains in the concrete and steel. The agreement between test and theory was satisfactory.

Finally, the effects of varying the time–dependent material properties and the reinforcement ratio were used to study the variation of stresses in the concrete and steel. Generally, the effects of creep and shrinkage result in a decreasing of the compressive stress in the concrete, as too does increasing the reinforcement ratio. This decrease may be so great that tensile stresses develop in the concrete, particularly under self–weight only, but the tensile stress calculated of about 2 MPa is not large, and little or no cracking would be likely to develop. It is noteworthy, however, that the midspan stress in the top of the slab, although subjected to sagging bending, may in fact be tensile.

It was found that the effects of creep and shrinkage had a significant effect on the stress in the bottom fibre of the steel joist. However, the effect is unlikely to affect the design of such a beam as under a realistic sustained load as the stresses in the steel joist are well below yield.

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REFERENCES


NOTATION

The geometrical properties are shown in Fig. 1. Other principal notation is as below:

\[ A \] Transformed area according to \( n \)
\[ A_c \] Area of the concrete part of the cross-section
\[ \bar{A}_c \] Age-adjusted transformed area
\[ A_{sr} \] Area of reinforcement
\[ A_{ss} \] Area of steel joist
\[ B \] Transformed first moment of area according to \( n \)
\[ B_e \] First moment of area of concrete slab
\[ \bar{B} \] Age-adjusted transformed first moment of area
\[ E_c \] Short-term elastic modulus of concrete
\[ \bar{E}_e \] Age-adjusted modulus of concrete
\[ E_s \] Elastic modulus of steel
\[ I \] Transformed second moment of area according to \( n \)
\[ I_c \] Second moment of area of concrete slab
\[ I_e \] Age-adjusted transformed second moment of area
\[ I_{ss} \] Second moment of area of steel joist about its centroidal axis
\[ M_i \] Short-term applied moment
\[ -\Delta M \] Restraining moment in Eq. 15
\[ N_i \] Short-term applied axial force
\[ -\Delta N \] Restraining axial force in Eq. 14
\[ n \] Modular ratio \( E_s / E_c \)
\[ p \] Slab reinforcement ratio \( A_{sr} / b D_c \)
\[ q \] Superimposed load in kPa
\[ y \] Coordinate from top fibre of section
\[ \varepsilon_i \] Short-term strain
\[ \varepsilon_{oi} \] Short-term strain at top fibre of section
\[ \Delta \varepsilon_c \] Time-dependent change in top fibre strain
\[ \varepsilon_{sh} \] Shrinkage strain
\[ \rho_i \] Short-term curvature
\[ \Delta \rho \] Time-dependent change in curvature
\[ \Delta \sigma_c \] Time-dependent change in concrete stress
\[ \Delta \sigma_{sr} \] Time-dependent change in reinforcement stress
$\Delta \sigma_{ss}$ Time-dependent change in joist stress
$\phi$ Creep coefficient
$\chi$ Aging coefficient
FIG. 1 COMPOSITE CROSS-SECTION
FIG. 2 COMPOSITE BEAM TESTED
FIG. 3 RESULTS OF SHRINKAGE TESTS
FIG. 4 CREEP COEFFICIENT

- Tests
- Best fit

Loading at $t = 10$ days

Creep coefficient $\phi$
FIG. 5  STRAINS AT TOP OF SLAB
FIG. 9  STRESSES AT SOFFIT OF SLAB
$E_s = 200 \times 10^3$ MPa

$E_c = 25 \times 10^3$ MPa

FIG. 10  STRESSES IN BOTTOM FIBRE OF STEEL
FIG. 11  TOP OF SLAB STRESSES vs REINFORCEMENT RATIO
FIG. 12  SLAB SOFFIT STRESSES vs REINFORCEMENT RATIO