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THE DESIGN OF FLAT SLAB STRUCTURES – AN HISTORICAL SURVEY

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THE DESIGN OF FLAT SLAB STRUCTURES - AN HISTORICAL SURVEY

by

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SUMMARY

This paper traces the development of design methods for flat slab structures, and considers the current state of knowledge relating to the design of such structures for both vertical and lateral loads.

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CONTENTS

1. INTRODUCTION

This paper examines published work on elastic bending moments in flat slab structures. It traces the development of design methods and considers the current state of knowledge of flat slab behaviour.

Section 2 describes the rather stormy beginnings of flat slab construction in the first two decades of the twentieth century. The early controversies are of considerable historical interest and their effects may still be discerned in the provisions of today's building codes.

Investigations into the bending moments produced by vertical loads are considered rather briefly in Section 3, since this area has been well surveyed by other investigators. Published work which is directed towards stiffness properties and design for lateral loads is examined in more detail in Sections 4 and 5.

2. EARLY HISTORY 1905 - 1921

2.1 Early Construction

The advent of reinforced concrete allowed the extension of structural design from one-dimensional to two-dimensional elements. The development did not come easily. One of the pioneers of flat slab construction, Robert Maillart, wrote later:

"With these one-dimensional elements: joists, columns and beams, the engineer was accustomed to calculate and build his structures, in such a manner that they were everything in the world for him and any other possibility lay far beyond his reach.....This was the situation when reinforced concrete emerged, but at first nothing was changed: it was laid as if it were steel or wood, girders spanned from wall to wall and from column to column. At right angles to these main girders came secondary beams and the space between would be filled in with a slab without however its being comprehended as a special constructional element. On the contrary they made haste to divide it up into strips, those strips could then be calculated as beams in the normal manner." (Ref. Bill, 1969).

A similar point was well made by Sozen and Siess (1963).

"Just as the first motor cars were built to look like horse-drawn carriages, the first reinforced concrete systems were conceived in the image of traditional types. In a timber structure, the planks carried the load to the joists, the joists to the girders, and the girders to the columns; so must they in a reinforced concrete structure. Hence, the flat slab had to be invented rather than developed as one of the obvious applications of reinforced concrete."

Credit for the invention goes first to the American engineer C.A.P. Turner. Writing in the "Engineering News" in 1905, Turner introduced his 'mushroom slab' thus:

"There is ample room for all to improve radically on present methods of design and computation which are....following after the manner of structural iron work.

"Thus far experimental investigation has been confined almost exclusively to simple beams and slabs reinforced in one direction only..... That concrete lends itself readily to reinforcement in all directions should lead the practical constructor, as far as may be, to so reinforce his work that the deformation from a strain in one direction may be offset in part by a force in another direction.

"Enclosed herewith (Fig.3) a study along this line, the idea being to avoid the expensive forms for beams to secure a neat and unbroken ceiling line together with a considerable economy of material without sacrifice of strength." (Ref. Turner, 1905).

The "study" referred to was probably the first structural drawing of a flat slab ever published. It is reproduced here as Fig. 1 and shows the four-way reinforcement system which was to characterise Turner's flat slab designs.

The first flat slab structure to be built was the five-storey C.A. Bovey-Johnson building, constructed by Turner in Minneapolis, Minnesota in 1906. Building Department permission was obtained for it only as an experimental structure subject to a load test, in which the floor was required to withstand an imposed load equal to almost three times the design load without deflecting more than 5/8" at the centre of any panel. Two adjacent panels were loaded; the maximum deflection was only ¼", easily satisfying the requirement.

This building was the first of many. In 1914 Turner was able to speak of "having introduced the mushroom system in about $200,000,000 worth of buildings and bridges, of spans from 12' to 50', in the past seven years...." (Ref. Eddy, 1914) and of his experience "acquired in the design and construction of from 1000 to 2000 structures of this type." (Ref. Nichols 1914).

The rapidity with which the flat slab system was accepted in the U.S.A. is emphasised by the fact that it was used for 80% of all buildings designed for loads of 100 psf or more in the period 1906-1913. The acceptance was not unqualified. Turner wrote in 1914: "The conservative business man who advances the money, as the writer has found by experience, would usually like a bond, which may amount to anywhere from $5000 to $100,000, to assure him that the structure when completed will come up to the guaranty." The "guaranty" involved tests of load carrying capacity and maximum deflection. Since the engineering profession was quite unable to agree on a method for flat slab design, and since the cost of a flat slab was known to vary markedly depending on which engineer designed it, the business man was undoubtedly justified in his conservatism.

During this period the Swiss engineer, Robert Maillart, was pioneering flat slab construction in Europe. Maillart's first step was to conduct experiments on large scale models. He wrote: "Of course it is a problem that is hardly solvable by calculation and so only tests with models and the measuring of executed buildings can lead to a more certain goal. "(Ref. Bill 1969, p.165).

Figures 2 and 3 show the first flat slab structures to be built for experimental purposes. They were constructed in 1908 in the workyard of Maillart and Cie in Zurich "in order to first obtain a foothold concerning the constructive possibilities of the reinforced concrete slab." The first test slab, shown in the foreground of Figure 2, was pin-supported at the four corners, and was quickly found to be unsuitable. The second structure, shown in Figure 3, consisted of a nine-bay slab rigidly connected to its columns via capitals, and "proved of such stiffness, even with point loads in single fields, that the practical value of the system was proved." This structure was claimed by Maillart to be the first flat slab to use a two-way system of reinforcement, rather than the four-way pattern favoured by Turner.

This test convinced Maillart of the practicability of the flat slab system. "The problem now was how could it be constructed and dimensioned. The purely theoretical way appeared to be inaccessible. .....In order to achieve an experimental basis for the problem, and to acquire a sufficient basis for practice, the same firm erected a large structure with nine fields each of four meters length. The slab was only 8 cm. thick to assure the greatest possible elasticity." This test structure, built in 1910, is shown in Figure 4. A single concentrated load of 1000 Kg was applied at various points on a one-meter grid; the deflection curves for a variety of loadings were obtained from the test readings by superposition. A number of simply supported beams were also made with the same thickness and reinforcement as the test structure. "The influence of the single loads could now be judged since the deflection curve of the slab could be compared with the deflection of the beams under the corresponding known bending moments." Although it seems unlikely that much reliable information on moments or stresses could be obtained in this way, the tests gave Maillart sufficient confidence to proceed with commercial construction of flat slab buildings.

The first of these buildings was the Lagerhaus-Gesellschaft building, constructed in Zurich in 1910. In addition to using two-way rather than four-way reinforcement, Maillart's system differed from Turner's in employing a curved column capital (see Fig. 5) so that "the column fuses into the floor slab corresponding to the play of forces." No information has been found on the methods by which Maillart determined the slab thickness and reinforcement quantities required.

2.2 Early Theory and Design Methods

The emergence of the flat slab system in the United States aroused intense controversy among structural engineers. Sozen and Siess (1963) comment: "For the structural engineer, plate action was an entirely new concept. The 'crossing beam analogy' thinking of the slab as two perpendicular beams each carrying a certain proportion of the load in relation to their stiffnesses, helped only to foster the still existing illusion that only part of the load need be carried in a given direction. Grashof's work had already been used by the mechanical engineers in boiler plate problems. However, this work was represented in American engineering literature either simply as formulas without any derivations or as a basis for arriving at questionable conclusions."

The "questionable conclusions" usually involved a misunderstanding of what Mensch called "the mystic influence of Poisson's ratio" (Ref. Eddy, 1914, discussion). The chief authority for this school was Professor Eddy of Minnesota, who developed an elaborate theory of slab action from the basic assumption that the lateral strain due to Poisson's effect must be accompanied by a corresponding lateral stress. His theory naturally came into conflict with the laws of statics but it was popular among some of the "commercial designers", notably C.A.P. Turner, since it had the effect of appreciably reducing the negative reinforcement required over the columns.

Some engineers (then as now) doubted whether a structural system as novel and complex as the flat slab really came within the gamut of the laws of equilibrium. Professor Eddy wrote of "the essential divergence of the correct theory of slab action from that of beam action in which latter case .....the moment of the applied forces is equal to the moment of the internal resistance, which is not true of slabs." (Ref. Nichols, 1914, discussion).

Eddy's theories did not go unchallenged. E. Godfrey complained that "In asking the Profession to accept this mushroom theory, the author (Eddy) is asking us to consider the laws of matter suspended for this particular type, or else that there are special laws that apply to this special combination of materials." (Ref. Eddy, 1914, discussion).

The practical effects of the controversy were highlighted by Angus B. McMillan in 1910. In an article in the Engineering News (Ref. McMillan, 1910), he described six methods then in use for the design of flat slabs. The methods fell into two broad classes: those based on (or misconstrued from) Grashof's theory, and those using one of the variations of the 'cantilever method'. In the latter method a rectangular or circular section of slab surrounding the column head and bounded by the inflection lines, usually assumed at about one-fifth of the span from the column centre-lines, was assumed to act as a cantilever, supporting the loads on its surface and a concentrated line load at the perimeter which supported the rest of the slab.

McMillan applied each of the six methods to the design of a 20ft x 20 ft interior panel of a flat slab carrying 200 psf live load, and tabulated the resulting slab thickness and reinforcement quantities per panel. His table is reproduced below. Methods 1 and 2 used the cantilever approach, methods 3, 4, 5 and 6 were deduced from Grashof's work.

Design Method	Slab Thickness (in.)	Steel Stress (psi)	Amount of Reinforcement per panel (lb)
1. Cantilever	8	16,000	2,189
2. Turneaure & Maurer	12	16,000	1,931
3. Grashof	8	16,000	784
4. Mensch	8	16,000	2,120
5. Turner* (a)	8	16,000	549
(b)	8	13,000	718
6. McMillan	8	16,000	1,084

*Turner used and recommended a steel stress of 13,000 psi

(a) was included for purposes of comparison.

The table indicates that the quantity of reinforcement used could vary by as much as 400% depending on which design method was used. Turner's method gave the smallest quantity of all; small wonder that his clients demanded bonds dependant on satisfactory performance of their buildings in load tests.

The differences between design methods were considerable; the financial stakes for some of the designers were huge. (Turner claimed to have used his system in two hundred million dollars' worth of structures over a seven year period.) The debate between the 'commercial' designers and their more cautious brethren was bitter and public. The discussion of a paper by Professor Eddy highlighted the attitudes of the opposing factions (Ref. Eddy, 1914).

Godfrey complained that "the commercial designers succeed in scaling down the bending moments to a point which independent practising engineers fear to attempt.....The theory of flat slabs is hard for any disinterested engineer to accept." L.J. Mensch wrote of "A riotous licence of figuring by most of the advocates of flat slab construction.....The writer begs not to be misunderstood as being an opponent of flat slab construction...He protests, however, when engineers and contractors (who, as a rule, have all to gain and nothing to lose) represent by versatile agents that flat slab construction, of designs as advocated by Mr. Eddy, are as good as, or better than, girder constructions which actually have a factor of safety of four or more....No wonder that they can show to unsuspecting architects and owners a great saving over all other designs, and being able to mention a great number of examples of buildings which did not fall down (having a factor of safety of about 2) are believed to be by the owners and architects very wizards in the art of reinforced concrete construction."

C.A.P. Turner, on the other hand, was able to reply by invoking the prestige of a large successful practice: "Having introduced the mushroom system in about $200,000,000 worth of buildings and bridges, of spans from 12 to 50 ft, in the past seven years, the writer may state that more testing has been done and larger bonds written guaranteeing its strength than for any other kind of concrete construction.....when it is considered that it has been put up in Australia, India, the West Indies, throughout Canada and the United States, the record of achievement must have behind it something more than mistaken ideas."

2.3 Early Tests

The weight of evidence was on Turner's side.

Flat slab buildings, no matter now designed, passed their load tests with ease. Furthermore, evidence was accumulating from more sophisticated tests in which reinforcement strains were measured.

The first such test was made in 1910 by A.R. Lord, who measured reinforcement strains on a flat slab floor of the Deere and Webber building in Minneapolis. The 9-3/16" thick slab was designed for a live load of 225 psf, and in the load test eight adjacent interior panels were subjected to an imposed load of 350 psf. The maximum measured negative and positive moment stresses were 24 and 10.4 Ksi respectively.

This was the first of several similar tests on flat slab buildings constructed in the U.S.A. in the period 1910-1920. The tests have been summarised in a thesis by D.S. Hatcher at the University of Illinois (Ref. Hatcher, Sozen & Siess, 1961). In all cases uniform load was imposed on a number of panels (varying from one to nine) of a large multi-panel floor. Steel strains were measured using extensometers with gauge lengths varying from 8" to 15", and the usual straight-line reinforced concrete formula, $M = f_s A_s jd$, was used to calculate the bending moments from the strains. In all tests the strains measured at working loads, and therefore the moments deduced from them, were found to be small.

2.4 Nichols, 1914

The flat slab debate appeared to be deadlocked. Turner, who claimed to base his calculations on Grashof's theory, designed his flat slab panels for a total negative moment over interior columns of WL/50. McMillan, also using Grashof's theory, concluded that the correct figure for this moment was WL/25. The proponents of the cantilever method doubled the figure again, and used steel quantities four times greater than Turner's. Yet Turner's slabs stood, and tests indicated that steel stresses at working loads were well within safe limits. Perhaps the only point of agreement among the disputants was that flat slab analysis was complex and "hardly solvable by calculation'.

In 1914 a young Boston engineer, J.R. Nichols, broke the deadlock by showing that the total moment for one special but commonly occurring case could be calculated from the equations of statics.

Nichols considered an interior panel of a regular flat slab floor with square panels extending indefinitely in both directions, supported on circular column capitals and uniformly loaded (see Figure 6). The segment shown in heavy outline, bounded by sides A,B,C,D and E, was taken as a free body.

Due to symmetry, no shears or twisting moments exist on faces B,C, D and E. There are bending moments on all faces, and shears and twisting moments on face A. The assumption is made that the shear forces on the curved face A are uniformly distributed.

The total vertical load on the free body is $0.25w(L^2 - \pi r_o^2)$, which must also be the value of the total shear on face A. The resultant of the shear forces, assumed uniformly distributed around A, acts at a distance from the centre of the column equal to $2\sqrt{2}r_o/\pi$.

Taking moments of all vertical forces about XX gives:

$$M_{xx} = \frac{wL^3}{16} (1 - 2.55k + 2.67k^3) \text{ where } k = r_o/L$$

For equilibrium, this must be equal to the sum of the bending moments on faces C and B, plus the components normal to XX of the moments on the curved face A.

In his conclusion to the discussion on his paper, Nichols suggested a simpler approximation to the above formula. Extending the free body across the full width of the panel, the approximate expression for total moment is

$$M_o = \frac{WL}{8} (1 - \frac{2}{3}\frac{c}{L})^2$$

where W is the total load on the panel, and c the diameter of the column cap. The error involved in the approximation is less than 1% for values of c/L smaller than 0.3.

Obviously, Nichols' formula did not enable the moment at any point, or across any section, to be determined; but it did provide the first sound criterion against which designs could be checked. Nichols himself commented: "The nature of the limitations imposed by statics is best shown by an illustration. If we are told that three stones weigh 6 lb. this does not establish the weight of any one stone. but it does ensure that the heaviest stone weighs at least 2 lb."

Nichols had done little more than apply the equations of statics to a flat plate panel, yet his paper provoked a furious discussion, five times as long as the original paper.

A.W. Buel complained that "the author's reasoning seems to be deductive, which since the time of Lord Francis Bacon, has not been in favour for scientific investigation." L.J. Mensch thought that Nichols' paper "will not advance our knowledge of the design of such floors, as he assumes a certain relation of positive and negative moments, and fails to prove that they may exist."

C.A.P. Turner, leaning confidently on his experience "acquired in the design and construction of from 1000 to 2000 structures of this type, dismissed Nichols' thesis as "mere algebraic deductions which the author has based on certain assumptions. These assumptions and deductions by Mr. Nichols appear to involve the most unique combination of multifarious absurdities imaginable from either a logical, practical or theoretical standpoint. At the very outset he assumes the illogical proposition that the mechanics of a slab and the mechanics of a beam are identically the same." He invoked the authority of Professor Eddy, who "clearly defines the external moment of forces acting on a slab or beam as apparent moments."

Eddy joined the attack with an exposition of his flat slab theory, which, as noted earlier, was based on an erroneous treatment of Poisson's ratio effects. He further compounded the error in a tortuous attempt to justify the use of a Poisson's ratio value of 0.5 for reinforced concrete. This had the effect of halving the magnitude of the calculated negative moments over the columns, an achievement which suited Turner and appeared to give closer agreement with reinforcement stresses measured in building tests. The agreement was in fact fortuitous, and it was perhaps unfortunate that Eddy chose to congratulate himself in this discussion on "having brought .... a rational theory to a somewhat satisfactory degree of perfection."

The forces arraigned against the application of Newton's laws to flat slabs must have seemed formidable. G.S. Binckley, in discussing a paper by Eddy in the same year, wrote: "The crushing weight of practical experience under which C.A.P. Turner, M.Am.Soc.C.E., flattened out Mr. Nichols' purely theoretical paper tends to induce caution in others."

2.5 Early Code Requirements

The available empirical evidence appeared to support Turner's 'experience'. His slabs used much less steel than Nichols' analysis would have required, yet they performed satisfactorily. Furthermore steel strains had been measured in several buildings, and the total panel moments deduced from them were in all cases much smaller than the value of M_o calculated from

Nichols' formula. For example, tests on six flat slab floors yielded the following values of total moment:

Purdue test slab J	$0.59\ M_o$
Purdue test slab S	$0.74\ M_o$
Western Newspaper Union	$0.72\ M_o$
Sanitary Can building	$0.30\ M_o$
Shonk Building	$0.38\ M_o$
Bell St. Warehouse	$0.40\ M_o$

The engineering profession was thus faced with a conflict between Newton's laws and apparently overwhelming test evidence in favour of "suspending the laws of matter" for flat slabs. Unable to ignore either Nichols' logic or the test figures, it adopted the form of Nichols' equation, but arbitrarily reduced the magnitude of the total moment. The first Joint Committee decided that flat slabs should be designed for a total moment of

$$M_o = 0.107WL(1 - \frac{2}{3}\frac{c}{L})^2$$

representing 85% of the static moment. The 1920 A.C.I. Code rejected the conditions of equilibrium even more decisively with;

$$M_o = 0.09WL(1 - \frac{2}{3}\frac{c}{L})^2$$

thereby authorising a disregard of statics which persists in building codes to this day.

2.6 Westergaard and Slater, 1921

Nichols' paper had shown how the total moment in an interior panel could be calculated. It gave no information on the distribution of moments within a panel, and it did not explain the low stresses measured in building tests. These problems were resolved when Westergaard and Slater (1921) published a comprehensive paper which aimed "to present information which correlates the results of tests of a fairly large number of slab structures with the results of analysis, so that the report may aid in the formulation of building regulations for slabs."

The first section of the paper, written by Westergaard, dealt with the analysis of homogeneous elastic plates. It commenced with an historical summary of the development of plate theory, covering 42 references from Euler (1766) to Nielsen (1920), followed by a derivation of the plate equation,

$$\frac{\partial^4 z}{\partial x^4} + \frac{2\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4} = \frac{1-\mu^2}{EI} w$$

and a comprehensive discussion of bending moments in rectangular plates supported on four sides.

Westergaard then considered a typical interior square panel in a regular flat slab structure of infinite extent, supported on rigid circular columns, and uniformly loaded. A solution was achieved by first using results obtained by Nielsen from finite difference analyses of slabs on point supports, so that the combined solution gave zero slopes and deflections around the peripheries of the columns.

These analyses yielded the first reliable information on the distribution of elastic moments in a flat slab panel. The results for c/L ratios varying from 0.0 to 0.3 were given in two figures which are reproduced here as Figures 7 and 8. It was found that the proportion of the total static moment taken by the various parts of the panel did not vary significantly with varying c/L ratio. Average values were:

	Column Strip	Middle Strip	Total
Negative Moment	48%	17%	65%
Positive Moment	21%	14%	35%

These percentages are very close to those used in today's building codes.

In addition to this work on uniformly loaded interior square panels, Westergaard also obtained some limited information on the effect of uneven panel loading, and on moments in oblong and exterior panels.

In the next section of the paper, Slater examined the relationship between reinforcement stresses and bending moments in beams and slabs. He was able to show the inadequacy of the assumption, made in previous analyses of flat slab tests, that moments could be obtained from measured steel strains by simply substituting in the straight-line formula $M = E_s \varepsilon_s A_s jd$. Slater commented:

"In a cracked beam the stresses at the cracks may approach the computed stress, but between the cracks the concrete assists so greatly in carrying the stresses that the average measured unit-deformation over the gauge length is likely to be considerably less than the maximum unit-deformation, especially at the lower loads. It is possible also that even at the section where a crack occurs a portion of the moment may be resisted by the tensile stresses in the concrete."

Slater investigated the relationship between measured steel strains and bending moments by testing 84 beams with varying steel percentages. He compared the steel stresses indicated by measured steel strains with the stresses calculated from $f_s = M/A_s jd$, and found that the measured stresses were significantly smaller than the calculated values for all loads up to the ultimate. With low steel percentages and low loads, the measured stress could be as little as one-quarter of the predicted value at working loads.

These tests exposed the major reason for the differences between the static moments and those deduced from steel strains in the 1910-1920 building tests. All of the strain measurements were made with long gauge length extensometers; steel percentages in the slabs were low; concrete tensile stresses were therefore very significant. There were other contributing reasons - the effect of adjacent unloaded panels, the neglecting of twisting moments around the large column capitals of these buildings - but errors due to these causes would have been small compared to those introduced by ignoring concrete tension. Slater's work brought about the major reconciliation between Turner's flat slabs and Newton's laws.

The 1921 paper by Westergaard and Slater was a monumental work which remains perhaps the most important contribution yet made to knowledge of moments in slabs under vertical loading. It marked the end of the beginning of the search for a practical solution to the flat slab problem.

3. VERTICAL LOADING

3.1 Theoretical

3.1.1 Interior bays of Uniformly Loaded Floors

If a flat slab floor has a regular array of rectangular bays in both directions, is uniformly loaded over its entire area, and is so large that for practical purposes an interior bay may be considered to be surrounded on all four sides by an infinite number of identical bays, then the two centre-lines of the interior bay, and the four column centre-lines that bound it, are all lines of symmetry on which no shears or twisting moments exist. The resulting simplification of the boundary conditions makes this a special case for which the complexities of flat slab analysis are greatly reduced.

As discussed in the previous section, this case was first considered by Nichols (1914) who derived an expression for the total static moment in the bay, and by Westergaard and Slater (1921), who calculated the distribution of bending moments within an interior bay of a flat slab structure supported on circular column capitals. Westergaard and Slater dealt mainly with square bays, although they obtained some limited information on moments in oblong bays, with aspect ratios up to 1:1½.

An approximate solution for square interior bays supported on square column capitals was published by Woinowsky-Krieger (1954), who used a complex variable method in conjunction with conformal mapping, originally developed by Muschelisvili for two-dimensional and torsional problems in elasticity. The method was laborious, and its use was made practicable only by the high degree of symmetry obtaining in this special case. The paper presented graphically the bending moment distributions calculated for a slab with column side: bay span ratio c/L = 0.2.

Gupta and Vaughan (1967) extended Woinowsky-Krieger's work to include rectangular bay slabs supported on rectangular columns. The theoretical solution was checked against experimental results obtained from a small scale multi-bay Perspex model. Curvatures in the interior bay of the model were measured by the moire fringe method, and showed good agreement with calculated values.

3.1.2 General Theoretical Solutions

An ingenious mathematical solution was published by Brotchie (1957 and 1959), who observed that the elastic analysis of flat plate structures was made complex by the interaction of loads and reactions. The loads and reactions individually were simple in form or could be divided into simple components; the complexities of analysis could therefore be reduced by considering the components one at a time, and combining them by superposition.

This was achieved by supporting the slab on an imaginary liquid, and applying each load or reaction component separately. The reaction provided by the liquid was proportional to the deflection of the slab and could be either positive or negative, since the liquid was assumed to remain in contact with the slab at all times.

Polar coordinates were used, and values of radial and tangential moments were tabulated for (i) point loads and (ii) a uniform radial moment applied round a circle of small radius. By combining the two cases, and by numerical integration from the point loading case, the effect of any type of load on a flat slab supported on circular columns could in principle be determined. However, the amount of work involved made the use of the method rather unattractive except for very simple cases.

Russell (1966) used Brotchie's method to produce graphs of the distribution of moments in interior bays under uniform load, for oblong bays with aspect ratios varying from 1:1 to 1:3.

Brotchie (1963) later modified classical plate theory to examine the effect on bending moments of the in-plane or arching forces resulting from the extension of the middle surface of a slab due to flexural cracking. He concluded that if the boundaries of an interior panel were fully restrained, the in-plane forces could reduce the total moment by 17-20%. but that in a flat plate structure with no marginal beams, or with all bays loaded, the restraint largely disappeared. and the full static moment given by Nichol's expression should be used.

A numerical procedure for the analysis of rectangular plates supported on rigid columns of rectangular cross-section was developed by Ang. (1959) at the University of Illinois. He used Newmark's plate analogue to enable finite difference equations to be applied to the analysis of plates. In order to reduce the amount of computer storage required, Ang developed a distribution procedure which allowed continuous plates to be analysed by the interconnection of single panel solutions. The procedure was based on the Hardy Cross moment distribution technique for beams and frames, and commenced with the "fixed edge moments" and "fixed edge reactions" at all edges of all panels resulting from the loads applied on the panels. It consisted of (i) successively balancing moments at each joint common to two panels, around all joints in all edges, while maintaining continuity at the joints, and repeating the moment-balancing process cyclically until satisfactory convergence was obtained; (ii) similarly balancing one vertical reaction at a time at each joint common to two panels, while maintaining continuity, and repeating till the unbalanced reactions at all joints were reduced to acceptably low values; (iii) since the equilibrium of moments attained in step (i) was unbalanced by step (ii), (i) had to be repeated, thereby re-establishing equilibrium of moments but unbalancing the vertical reactions again... and so on. Clearly many cycles of computer calculations were required for the solution of general plate problems by this method. However, in the case of continuous plates in which all panels were identical, the balancing of the vertical reactions did not affect moment equilibrium at the joints, and the solution was obtained much more rapidly. Ang's work was restricted to this case.

Woodring and Siess (1968) combined Ang's distribution technique with a procedure proposed by Newmark (1941) in order to obtain influence surfaces for continuous plates supported on rigid rectangular columns.

Several charts showing influence surfaces for a 3 x 3 square bay structure were included in this paper.

3.2 Design Methods and Building Code Requirements

3.2.1 Introduction

The Americans were the original pioneers of flat slab construction, and have ever since led the way in devising and improving flat slab design methods. Australian and British Codes of Practice have consistently followed A.C.I. Code provisions in all essential points. Therefore only the A.C.I. Code will be discussed here.

3.2.2 The Empirical Method

An outline has been given in Section 2.5 of the circumstances which led to the adoption in the 1920 A.C.I. Code of a provision allowing flat slab panels to be designed for a total moment 28% smaller than the static moment. This Code also provided that, for slabs without drop panels, the minimum percentages of the total moment to be resisted by the various sections of the slab should be:

	Negative Moment	Positive Moment
Column Strip	40	18
Middle Strip	10	12

The remaining 20% was left for the designer to distribute "as required by the physical details and dimensions of the particular design employed."

The 1928 A.C.I. Code left the value of the total moment unchanged but revised the distribution percentages, following Westergaard's work, to:

	Negative Moment	Positive Moment
Column Strip	46	22
Middle Strip	16	16

The distribution coefficients, and the expression for total moment, remained unchanged in A.C.I. Codes until 1971, except that the 1963 Code increased the design value of the total moment by up to 15% for values of c/L less than 0.15.

The design of flat slabs by the use of the A.C.I. formula for total moment and coefficients for distributing that moment, was referred to as "Design by Moment Coefficients", or "Design by Empirical Method". The 1928 Code introduced a provision limiting the use of this method to structures similar to those which had provided the test data by which it was justified, i.e. to flat slab floors containing "a series of slabs of approximately uniform size arranged in three or more rows of panels in each direction, and in which the ratio of length to width of panel does not exceed 1.33." Since many flat slab structures fell outside these limitations, a design method which could be more generally applied was clearly needed.

3.2.3 Equivalent Frame Methods

In 1929 the committee charged with formulating the reinforced concrete section of the Uniform Building Code, California edition, set up a sub-committee to investigate the possibility of treating the flat slab and its supporting columns as a series of elastic frames. The sub-committee's report, published later by Dewell and Hammill (1938), led to the inclusion of an "Elastic Frame" method of flat slab analysis in the 1933 Californian edition of the Uniform Building Code.

The method adopted was very similar to equivalent frame methods in use today. A full panel width of slab constituted the "beam" of the bent. Column-slab joints were considered to be rigid. Columns were assumed to have points of contraflexure at their mid-heights. The positive and negative moments calculated from the frame analysis were distributed between column and middle strips in the same proportion as was specified for the Empirical Method.

One of the problems faced by the sub-committee was that, since the elastic frame analysis conformed to the conditions of equilibrium, it led to considerably larger moments than the Empirical Method, which accounted for only 72% of the static moment. This inconsistency was removed by the simple expedient of reducing by 40% the negative moments calculated in the frame analysis.

An equivalent frame method for flat slabs was introduced into the A.C.I. Code in 1941. It followed Dewell and Hammill's method closely, the main differences being that columns were considered to be fixed at their remote ends, and that the negative moment reduction was achieved by specifying that for design purposes the maximum negative moment should be taken as that obtaining at a specified distance from the column centre-line, this distance being devised so that the resulting design total moment for interior panels under uniform load was closely equal to that given by the Empirical Method formula. The specification of the critical section for negative moments was modified in the 1956 Code, but the effect remained substantially the same.

Corley, Sozen and Siess (1961) compared the moments calculated from the 1956 A.C.I. equivalent frame analysis with some known elastic solutions. It was found that, generally, the equivalent frame method gave values of positive moment which were too low, and values of negative moment (before reduction to critical sections) which were too high. After reduction the negative moments could be either high or low depending on the dimensions of panel and column. Usually the negative moments after reduction tended to be too low, except at exterior columns, where they were too high. In some cases the sum of the positive moment and the average of the reduced negative moments could be smaller than the value of the total panel moment given by the Empirical Method formula.

The reason for the high initial negative moments given by the equivalent frame method lay in the Code assumption that the slab-column joint was infinitely stiff. Even if the column itself were infinitely stiff the slab on either side of the column would undergo curvature. The error was magnified at exterior columns because the Code in effect assigned infinite torsional rigidity to the edge beams, by assuming the equivalent beam to be infinitely stiff within the limits of the column.

To improve the accuracy of the equivalent frame analysis for vertical loads, Corley proposed the following changes in the methods of calculating the stiffness of the ersatz members:

The stiffness of the slab within the slab-column joint, instead of being assumed infinite, should be taken as that of a slab twice as deep as the real slab.

The stiffness of an interior column in a flat plate structure should be taken as infinite within the joint. For flat slab structures with column capitals, the 1/EI diagram for the column should be assumed to vary linearly from zero at the mid-depth of the slab, to the 1/EI value for the column at the base of the capital.

At exterior columns, the edge beam-column should be considered as a single element whose average stiffness K_{bc} was to be calculated thus:

$$K_{bc} = \frac{m_1}{\theta_f + \theta_t} \qquad \text{where}$$

K_{bc} = stiffness of the beam (or slab) - column combination

m_1 = a distributed torque applied along the axis of the edge member

θ_f = rotation of end of column due to bending in column

θ_t = average rotation, due to twisting, of beam with respect to column.

In calculating θ_t, the torque applied by the slab was to be assumed to be linearly distributed along the edge member. Where there was no edge beam, a portion of the slab equal to the width of the column was to be considered to offer torsional resistance.

Moments computed by the proposed method were compared with moments measured in tests on five models, and in almost every case showed better agreement with the measured moments than did moments calculated using the 1956 A.C.I. equivalent frame method.

3.2.4 The 1971 A.C.I. Code

The 1971 A.C.I. Code incorporated significant changes in flat slab design rules, based largely on work carried out at the University of Illinois in the 1960's.

Two design methods were still specified: the equivalent frame method, and the "direct design method", which was based on the former "empirical method", but had less restrictive dimensional limitations so that it could be applied to a greater range of structures; the panel length:width limitation was extended from 1.33 to 2.0, and the ratio of successive spans from 1.2 to 1.33.

The formula for total panel moment, which had endured basically unchanged for fifty years, and which had been much criticised for its failure to account for the full static moment, was abandoned in favour of;

$$M_o = \frac{w L_2 L_n^2}{8}$$ where

w = applied uniform load per unit area

L_2 = span transverse to direction for which moments are being calculated

L_n = clear span between column faces

If the column reaction were assumed to be concentrated at the corners of the column, the total moment for an interior panel would be;

$$M_o = \frac{w}{8} (L_2 L_n^2 - c_1^2 (L_2 - c_2))$$

where:

c_1 = column dimension in direction of span

c_2 = column dimension in transverse direction

The code formula is an approximation to this expression, and is accurate to within 5% for values of c_1/L_1 smaller than 0.20.

Concentrating the reaction at the column corners is the most unconservative assumption that can be made in deriving the total moment. Perhaps the code committee, like its predecessor in 1921, was unwilling to go too far in one step beyond existing engineering practice. Using the more conservative assumption of shear forces distributed uniformly around the column perimeter the expression for total moment would be;

$$M_o = \frac{wL_2L_1^2}{8}\left(1 - 2\frac{c_1}{L_1} + \frac{c_1^2 c_2}{L_1^2 L_2}\right)$$

For square columns and square bays with c/L = 0.25 this expression gives a value of M_o about 12% greater than the code formula. The difference decreases with column size, and since the truth probably lies somewhere between the two extreme assumptions about the distribution of shears around the column, the error involved in the code formula should be small for modern flat plate structures.

The new code stipulated that for internal panels 65% of the total moment should be assigned to the negative moments, rather than the previous code's 62%. For exterior panels the distribution was governed by a formula which took into account the stiffness of the "equivalent column" in a manner similar to that proposed by Corley. The division of the positive and negative moments, into column strips and middle strips was essentially unchanged for interior panels, but for exterior panels without edge beams 100% of the negative moment was allocated to the column strip, on the basis of analytical investigations into the elastic distribution of moments in flat slab floors.

The equivalent frame method in the 1971 Code followed the principles of Corley's proposals, which were outlined in the previous section. The most important change made was to extend to interior columns also Corley's device of calculating column stiffness on the basis of the effective stiffness of a slab-column combination in which the strip of slab between the columns could rotate relative to the columns. This was found to be necessary in order to allow more accurately for the effect of pattern loads on positive moments.

The critical section for negative moments was now taken as the face of the column. Moments were distributed between column and middle strips in the same proportions as for the direct design method.

The revisions featured in the 1971 A.C.I. Code have made the design of flat slabs rather more complex, but there is no doubt that design moments calculated under the new provisions reflect much more accurately the elastic distribution of moments for slabs under vertical loading.

3.3 Experimental

Any attempt to determine experimentally the distribution of bending moments in continuous flat slab structures requires a major outlay of time, money and laboratory resources, and few such attempts have been made. The experimental work which has been published has been aimed chiefly at checking the adequacy of design methods given in building codes.

Bowen and Shaffer (1955) developed an optical method called "photo-reflective stress analysis", or the "Presan method", for measuring curvatures in an acrylic sheet model. The method, although time-consuming and expensive, could be applied generally, but the results given in the report applied only to a square interior bay supported on circular columns with a c/L ratio of 0.063. These results showed general agreement with the distribution of moments given in the 1951 A.C.I. Code; the biggest difference found was that the measured column strip negative moments constituted just over 50% of the total panel moment, compared with the code's 46%.

Higgins and Lin (1956) reported tests on a flat slab model consisting of a 3/8" cast aluminium sheet with six 17" square panels arranged three by two. Air pressure was used to apply uniform load to one panel at a time, and readings from individual tests were superposed to give results for all panels loaded. The authors' main conclusion was that the positive moments at midspan were consistently higher than A.C.I. (318-51) moments, and that the code positive moments should be multiplied by factors varying from 1.4 for column strips in exterior panels, to 3.4 for middle strips in interior panels. However, it should be observed that the A.C.I. analysis assumed that the remote ends of the columns were fixed, whereas from the published diagram of the model it seems that its columns were closer to a pinned than to a fixed condition. This would increase the positive moments in the slab; the increase would tend to accumulate as individual test results were superposed to obtain results for multiple panel loading.

A major experimental investigation was carried out at the University of Illinois in the period 1956-1963. (Ref: Sozen and Siess (1963); Hatcher, Sozen and Siess (1961); Hatcher, Sozen and Siess (1965).) The investigation was initiated by the Joint ACI-ASCE Committee on Design of Reinforced Concrete

Slabs with the aim of obtaining information on the strength and behaviour of various types of multiple-panel floor slabs. Five test structures were made; of which the first two, the flat plate and the flat slab, will be considered here. Each structure comprised nine 5ft square panels arranged three by three and was designed in accordance with the empirical method of the 1956 A.C.I. Code. They were model structures, in the sense that the designs were for structures four times as large, with 20ft x 20 ft bays, and all dimensions were scaled down by multiplying by 0.25. Coarse sand was used as aggregate in the 1.75" thick slabs, and the reinforcement was cut from 1/8" square bars, annealed to give stress-strain characteristics similar to intermediate grade reinforcing bars and specially rusted to improve the bond performance. The flat plate was designed as a typical apartment building floor with a live load of 40 psf and an imposed dead load of 30 psf for partitions and finishes, giving a total design load of 155 psf. The flat slab was designed for 'light storage' loading of 200 psf plus dead weight 85psf giving 285 psf design load. Edge beams were incorporated in both structures, and the flat slab had 10" thick drop panels and column capitals at all except the corner columns.

In the design load tests the flat plate was hardly cracked, whereas the flat slab was cracked extensively both top and bottom. Steel stresses were generally about 4-5 ksi in the flat plate and 16-18 ksi in the flat slab. The differences in behaviour of the two structures at design load arose from the fact that the design load for the flat slab was about double that for the flat plate, although both used the same slab thickness. Clearly a large portion of the moment in the flat plate at this load was still being carried by concrete tension in the largely uncracked slab. It should however be noted that the modulus of rupture of the small aggregate concrete used was in excess of 600 psi, much greater than would be expected in normal structural concrete.

At a load of 225 psf on the flat plate, 45% higher than the design load, localised peak stresses up to 31 ksi were recorded at the interior columns, but steel stresses generally were only 15-18 ksi, still below the design working stress of 20 ksi. Evidently even at this load a considerable proportion of the moment in the flat plate was still being carried by concrete tension.

Slab moments were deduced from measured steel strains by means of a moment:steel strain relationship obtained from tests on concrete beams with very low percentages of reinforcement. The A.C.I. design moments for the interior panel were naturally concluded to be low, since the Code provided for only 72% of statics. Otherwise the tests showed reasonable agreement between measured and A.C.I. moments except in the vicinity of the spandrel beams. Design moments were lower than measured moments at the first interior column line and much higher at the exterior column line.

As a check on size effects in the University of Illinois quarter-scale models, a companion flat plate structure to 3/4 scale, with 15ft square panels, was tested by the Portland Cement Association Research and Development Laboratories in Skokie, Illinois (Ref: Guralnick and LaFraugh (1963)).

The behaviour of this structure under load was very similar to that of the University of Illinois flat plate. At working loads the plate was largely uncracked. In the centre panel the flexural rigidity appeared to remain that of the gross section up to a load of about 255 psf. In no case did the maximum steel stress at working load exceed half the design allowable stress of 20 ksi.

Deflections were proportionally a little higher than in the smaller model, presumably because the modulus of rupture of the concrete was lower. The P.C.A. structure also had more cracks than the ¼ scale model, which was to be expected since the bond properties of the ½" deformed reinforcing bars used must have been much superior to those of the plain rusted 1/8" square bars used at the University of Illinois.

The distributions of moments at working loads were very similar to those found in the ¼ scale model. The authors observed that the measured stresses in the negative reinforcement were much higher than those in the positive steel, and that positive moment cracking was slight relative to negative moment cracking, indicating a case for placing more of the total panel reinforcement in the column strips over interior columns.

It is interesting that the authors judged it still necessary, in 1963, to open their concluding remarks with the following statement:

"In view of the agreement found between measured moments at service load and the computed total static moments, it is confirmed that the equations of statics are applicable to flat slab structures, as they obviously must be."

Following these one-quarter scale and three-quarter scale concrete model tests, R.C. Elstner (1970) tested a 1/14th scale methyl-methacrylate plastic model of the same structure, in order to explore the usefulness of small-scale elastic models as a means of studying flat slab systems. The agreement between the results from this model and those from the University of Illinois tests was generally good, the moments measured on the plastic model being on average about 5% on the low side. Scaled deflections also agreed fairly well with the larger model.

4. LATERAL LOADING

Before 1960, no significant attempt had been made to assess the behaviour of flat plate buildings under the action of lateral loads. Blakey (1962) wrote:

"Flat plate structures are not usually used in tall slender buildings because it is believed that their lateral rigidity is inadequate unless special stiffening frames or shear walls are provided. Whether this belief is accepted or not, the occasion will arise when it is necessary to calculate the behaviour under lateral load of an unstiffened flat plate structure, and it has been found that this may also be done on the basis of elastic frame analysis. In this analysis the 'beam' in the frame is taken to be a strip of the slab equal to the width of the shear head if steel columns with grillage connections are used, or a strip equal in width to the column plus three times the thickness of the slab if concrete columns are used without shear heads. These strip widths are chosen to provide the best estimate of lateral deflection.

"In the analysis, relative stiffness factors may be calculated from the uncracked gross concrete sections for the 'beams'. The 'beam' moments should not be regarded as confined only to the width of the 'beam' strip, but may be added to the column strip moments."

Although these comments expressed views which were probably widely held among structural engineers at the time, no basis for them in theory or test has been found.

Beresford (1962) published some results from lateral load tests on an experimental lightweight concrete flat plate structure in Australia. The structure was not designed specifically for the lateral load tests, which constituted only one of several series of tests on it, and it was not well suited for providing information on the lateral load response of typical flat plate structures. It had three 12' x 9' bays in each direction, and consisted of a lightweight concrete slab supported on slender steel columns, with two types of slab-column connections; it was therefore a rather unusual type of flat plate structure.

Lateral loads of 3000 lbs were applied simultaneously at slab level to each of the four columns on one of the longer sides of the structure.

The main emphasis of the report on the tests was in comparing the experimental results with those obtained from an equivalent frame analysis, according to the S.A.A. and A.C.I. building codes, in which the slab stiffness was calculated on the basis of the gross moment of inertia of the full panel width.

Column moments, deduced from measured steel strains, were found to be 10% to 20% greater than those calculated. This was attributed to the fact that the uncracked section had been used in obtaining the slab stiffness.

In the attempt to find what proportion of the slab width, assumed uncracked, would yield the correct values of moment and deflection when used in a conventional rigid frame analysis, three such analyses were made, with assumed slab widths of 2', 6' and 12' (the full panel width). It was found that the width assumed made little difference, the resulting column moments being almost constant for the three cases; the calculated deflection increased by less than 15% as the slab width decreased from 12' to 2'. Beresford concluded that "the width of column strip which may be assumed is not critical as far as assessing column moments is concerned". Nevertheless he recommended that moments due to lateral loads should be determined using the column strip width only, and that the moments thus determined should be added to the column strip moments due to vertical loads.

The insensitivity of the column moments to the assumed value of the slab width reflected the fact that the steel columns used were extremely flexible relative to the slab. The columns were constructed from two 2½" x 2½" x ¼" boxed angles, having a compound moment of inertia of only 2.25in^4. Even for the smallest slab width used in Beresford's analysis (2') the "equivalent beam" EI was three times the column EI. Figure 9 shows that for EI ratios greater than two little change in moment or deflection occurs. Had the structures been built with concrete columns, say 9" x 9" with an EI value about twenty times as great as that for the steel posts used, the moments and deflections would have been very sensitive to quite small changes in the assumed effective width of slab.

An investigation into the restraint offered by the connecting slab to coupled shear walls in a regular multi-storey apartment building was reported by Barnard and Schwaighofer (1967). The building was effectively infinite in length. The shear walls occupied the entire width of the building except for the central corridor which comprised 1/9 of the building width. The longitudinal spacing of the shear walls was 4/9 of the building width. The authors wished to establish what width of slab could be considered effective in coupling the shear walls together.

The problem was investigated by testing a 22 storey model, consisting of two shear walls ¼" thick and 4" long separated by a 1" gap, and connected by a floor slab 1/8" thick, 9" long and initially 4" wide. Strains were measured at the two extreme fibres of one of the shear walls. Lateral loads were applied at every second storey.

No attempt appears to have been made to restrain the slope of the slab at its longitudinal edges, so that the relationship of the model tests to the real situation is a little uncertain.

The method used to investigate the effective slab width was to test the model with 4" wide slabs, then cut all the slabs to a smaller width, and test again. The smallest slab width tested was 13/16". The reasoning behind this procedure was that "if the entire width of the floor slabs is effective in coupling the shear walls, then any reduction in the width should lead to an increase in the wall stresses in the lower part of the structure".

The model test results showed that the extreme fibre stresses in the wall increases by 15% from 540 to 620 psi as the width of the slab was decreased by 80% from 4" to 0.81". This would seem to indicate that within this range the wall stresses were little affected by the slab width, and it is difficult to accept the authors' conclusion that the test results "show conclusively.... that the entire slab width is to be considered as effective in coupling the shear walls".

In the discussion, D. Michael of Ove Arup and Partners reported that his investigation of the same problem had shown that "the dominant dimension determining the effective slab width is the clear opening dimension between the walls. Up to a slab width equal to the clear opening the slab is almost fully effective. Additional widths of slab thereafter add decreasing amounts to the slab stiffness."

5. STIFFNESS PROPERTIES

5.1 Patel (1957)

The first known systematic attempt to gain information on the stiffness of flat plates was undertaken by M.N. Patel in a doctoral program, completed in 1957, whose purpose was "to study the interaction between a column and a flat slab due to an unbalanced moment at the joint caused by any external or internal load".

The problem was investigated experimentally by means of a 4' x 3' stainless steel model with 12" square bays and 2.7" dia., circular columns. The model had fifteen column positions, five in the longitudinal and three in the transverse directions; transversely the plate extended 6" beyond each outer longitudinal row of columns, the edges being left free. Fourteen of the column positions were permanently fixed, and consisted of steel washers clamped to a supporting plate. The central column joint on one of the transverse edges could be rotated by loading an attached cantilever arm. On this edge deflections were prevented but rotations were not restrained except at the fixed column positions. The edge therefore simulated a line of symmetry in a long building in which rotation was applied to a typical interior column joint.

The measured moment per unit rotation of this column joint was found to be $6.2Et^3/12$ in-lb. Doubling this value, and dividing by D, gives a non-dimensional stiffness coefficient $M/D\theta = 11.3$, applying for an interior slab-column joint with a c/L ratio of 0.225, or 0.20 if the circular columns used are converted to equivalent area squares.

Plate deflections were measured on a one-inch grid, but the accuracy of the measurements was rather poor.

In parallel with the experimental work, finite difference analyses were carried out. In these calculations, which were apparently done manually, all panel edges were assumed to be fixed, except those contiguous to the rotated column. Agreement between analysis and experiment was poor.

5.2 Brotchie and Russell (1964)

Brotchie and Russell quoted some results of calculations on the stiffness of square internal panels in flat slabs supported on circular columns and subjected to lateral loading. Details of the method of calculation were contained in the Part Two Supplement, Appendix 4, of the reference article.

The stiffness figures tabulated in this article are plotted here as Figure 10. It is important to note that the stiffness referred to is that of an interior joint in an infinite structure, when an entire transverse row of columns is rotated simultaneously.

5.3 Carpenter (1965)

5.3.1 Scope and Methods

In 1965, J. Carpenter presented in a doctoral thesis the results of an analytical and experimental study of the elastic behaviour (stiffness properties, distribution of moments and deflections) of flat plate structures subjected to lateral loads.

The analytical work was limited to the case of a typical square interior panel in a regular structure with square columns. The experimental work concentrated on the same case although some limited information was obtained on the behaviour of the plate around a centre column on the edge of the structure.

The structure considered was a flat plate floor consisting of four equal square bays in each direction. It was assumed that, if a rotation were applied to the centre column in this structure while the other 24 columns were held fixed against rotation, the behaviour of the plate around the centre column would be independent of the edge conditions of the structure. The floor was considered to be an intermediate floor in a multi-storey building, with the points of inflection in the columns at mid-height, so that the loading could be represented by a single force in the column stubs.

The experimental work was carried out on two Perspex models, which were identical except for the size of the column cross-sections. Model A had ½" x ½" columns, model B had 1" x 1" columns. The slab consisted of a ¼" Perspex sheet with four 9" x 9" bays in each direction. Columns were loaded by equal and opposite forces at the ends of 3" column stubs above and below the slab.

Both models were instrumented similarly. A profile plotter was used to measure deflections on a 14 x 14 grid in a panel. Surface strains were measured at about 30 locations associated with each loaded column, using linear electric resistance strain gauges aligned parallel to one or other of the column lines. Stresscoat brittle lacquer was used in an attempt to determine visually the direction of the principal strains in the vicinity of the columns.

5.3.2 Analytical Work

An analysis of the flat plate structure was attempted only for the case of rotation of the typical interior column. The mathematical model used was a fixed-edge circular plate with a radius equal to twice the column spacing. The centre of the plate had a rigid inclusion simulating the rotated joint. The inclusion problem was solved by an approximate method involving complex variable theory and conformal mapping. The solution process produced deflections at the eight other column positions included within the circular plate; these deflections were reduced to zero by applying point loads of appropriate magnitudes at the column corners.

A "reference structure" was defined with which the various properties of the flat plate structure could be compared. The reference structure was identical with the A.C.I. "equivalent frame" for elastic analysis, except that the slab and columns were not considered to be infinitely stiff within the joints.

5.3.3 Experimental Results

Analytical and experimental results were compared, for the case of rotation of the centre column, as a guide to the effectiveness of the mathematical model in simulating the behaviour of the physical model.

Stiffness

The values of M/Dθ for the two models are compared with the analytical predictions in the table below. For the purpose of this comparison the stiffness of the reference structure was adjusted thus:

$$\frac{M}{\theta} = \frac{4EI}{L} \qquad E = \frac{12D(1-\mu^2)}{t^3} \qquad I = \frac{L\,t^3}{12}$$

Hence $\frac{M}{D\theta} = 4(1-\mu^2)$

For an interior panel this value must be doubled.

	EXPERIMENTAL RESULTS	ANALYTICAL RESULTS	REFERENCE STRUCTURE
Model A ½" x ½" Columns	5.38	4.95	6.96
Model B 1" x 1" Columns	6.49	6.90	6.92

The predicted stiffness was 8½% less than the measured stiffness for Model A, and 6% greater than the measured stiffness for Model B.

In each model loading tests were performed on two central edge columns. One column had its outside face flush with the edge of the slab; the other stood proud of the edge by a distance equal to one half of the column dimension.

The measured stiffness of the panel containing the flush column was 82% of the interior panel stiffness for Model A, 96% for Model B. The measured stiffness of the panel containing the exposed column was 69% of the interior panel stiffness for Model A, 90% for Model B.

Carry-Over Factor

The experimental results indicated that almost all of the carried-over moment was taken by the first longitudinal column. The design of the experiment did not provide for straightforward measurement of the moment in the fixed column, and considerable difficulty was experienced in deciding a value for this moment. In fact the analytical and measured values of the carry-over factors varied by up to 30%, and there were obvious inconsistencies in the experimental values, as shown in the table below:

CARRY-OVER FACTORS

		MODEL A	MODEL B
	Interior Col. (Anal)	0.222	0.288
	Interior Col. (Meas)	0.170	0.244
Flush	Exterior Col. (Meas)	0.142	0.248
Exposed	Exterior Col. (Meas)	0.189	0.227

The carry-over factor was defined as the ratio of the moment produced in the first longitudinal fixed column to that fraction of the applied moment which tended to rotate the fixed column, this fraction being one-half for an interior column loaded, and unity for an exterior column loaded.

Plate Moments

The experimental values of plate moments were generally within about 20% of predicted values, where these were given for comparison. The distribution of moments in the immediate vicinity of the columns could not be obtained from either the experimental results or the analysis. No information was obtained on twisting moments.

5.3.4 Predicted Structural Behaviour of Interior Panels

In the final section of the thesis it was assumed that the derived analytical expressions correctly predicted the behaviour of interior panels around a loaded column. On this basis the conditions in an interior panel were investigated for two loading cases:

1) Central Column only rotated, with all other columns fixed.

This case, which has already been discussed, gave information on stiffness properties needed for frame analysis. Figure 11 compares Carpenter's results with those published by two other research teams. In examining this figure it should be remembered that all computed results apply to interior panels only. The comparison with Brotchie's findings is based on Brotchie's definition of stiffness in which an entire transverse row of columns is rotated simultaneously. The results from Khan and Sbarounis were taken from model tests on a single panel with a central column, with no provision for applying moments at the transverse edges.

The curve marked "Brotchie square" was obtained by considering as equivalent, square and round columns of equal area.

In this figure 'c_x' is one half of the column dimension in the direction of the span l_x. All curves drawn apply to square interior panels.

2) Equal Rotation Applied to All Columns.

This case is of interest in giving information on the distribution of moments in flat plate structures subjected to lateral loads.

All the work done on this case used a Poisson's Ratio of 0.35. The results were all presented as ratios of the behaviour of the reference structure Since the "equivalent beam" of the reference structure could not adequately take into account the fact that the stiffness of the plate structure depended on the column: span ratio c/L, or that the carry-over factors were quite different from those applying to a prismatic member, the use of reference structure served only to obscure the results, and detracted from the usefulness of this section of the thesis.

5.4 Copley (1966)

An interesting approach to the problem of the stiffness of a typical interior panel in a multi-storey flat plate building, when all columns are subjected to the same rotation, was contained in a thesis by J. Copley in 1966.

Two approaches were used in this thesis:

1. In a chosen multi-storey building, the assumed effective width of the slab was varied while column sizes and loading were kept constant, in order to probe how sensitive were the moments transferred between floors and columns to change in plate stiffness;

2. The deformation characteristics of a plate under the action of an applied moment were studied and compared with the deformation of a beam under the same loading, in order to arrive at an effective width for the plate. This study was limited to the case of typical interior bays in an infinite building.

1) Equivalent Frame Analysis of Multi-Storey Building

The building chosen for analysis was twenty storeys high and two bays wide. Columns were on 24' x 24' grid, and the floor to floor height was 10'0". The plate thickness was 10". Column cross-section sizes varied from 10" x 10" at the top to 25" x 25" at the bottom of the building for exterior columns, and from 10" x 10" at the top to 35" x 35" at the bottom for interior columns. Design wind velocity was 90 m.p.h., resulting in a horizontal load per floor of 5.84 kips. The design vertical live load was 100 psf.

This building was analysed by computer for five different assumed effective plate widths, viz., full plate width, and ½, 1/3, 1/6, and 1/12 of full plate width.

It was found that the changes in assumed effective plate widths caused only minor changes in the bending moments for floors above the 6th floor. The mean results were obtained for the assumption of 1/3 of full plate width, and this value was recommended for use for the upper floors of a building. Copley commented:

"For the worst consideration of vertical and wind loading the moments from widths B/2 and B/6 do not vary by more than 10% from the value for B/3. Since this is less than the errors expected in the estimate of loading, strength of materials, etc., this width could be taken for design purposes.

"However, for the lower six floors .... the effective plate width is important in design and a reasonably close estimate of its value is necessary."

2) Mathematical Analysis of a Plate

The effect of wind loading on the system was represented by an infinite plate supported on columns each of which exerted a moment on the plate. Initially the cross-sectional size of the columns was taken as zero.

The moment M applied by each column to the plate was replaced by a couple consisting of two point loads (each P) acting at a distance c from the column centre-line, as indicated in Figure 12.

Since the building considered was regular and infinite in extent, the lines x = 0 and x = a, were lines of contraflexure where the moment and the deflection were zero.

The analytical model therefore reduced to a rectangular plate of infinite length with simply supported edges and loaded by a series of point loads at a distance c from one edge.

The case of an infinitely long simply supported plate subjected to a concentrated load at a distance c from an edge has been analysed by Timoshenko; Copley was therefore able to obtain the solution for his analytical model by superposition.

For example, for Poisson's ratio = 0.1, B = 2a and c = 0.1a, the rotation of the plate at the column was found to be:

$$\theta = 0.431\frac{P.a}{E.t^3}$$

The slope obtained from beam theory, with the full width of the plate assumed effective ($I = 2a.\ t^3/12$) was:

$$\theta = 0.171\ \frac{P.a}{E.t^3}$$

Therefore for this case the effective plate width was obtained as 0.171/0.431 = 39.7% of the full width.

For other ratios of column spacing: span the results obtained were as shown in Figure 13.

It can be seen that the effective width of the slab for stiffness was fairly constant at about 40% of the span (not of the full plate width) for ratios of column spacing: span greater than 3/4.

The effective width expressed in terms of the full plate width (B) became very small as the ratio of full width: span increased beyond unity. This was because the beam stiffness increased directly with plate width, but the presence of the adjacent columns had very little effect on the stiffness at the centre column. For columns arranged on a square grid, the plate stiffness was only 1.1% greater than for the case where the column spacing was infinite.

The effect of using a column of finite width, instead of zero width, was then investigated. It was found that, for a square column grid, and a column width equal to 0.1a, the stiffness increased by 10% to $0.393P.a/E.t^3$. Since the increase in stiffness was small Copley recommended that it should be neglected.

5.5 Qadeer and Stafford-Smith (1969)

Information relevant to the stiffness of flat plates was contained in an article published in 1969 by Qadeer and Stafford-Smith.

This paper examined the resistance offered by a flat plate to rotation of shear walls, with the object of determining values for the interaction bending stiffness of the slab.

The structure analysed was an idealised regular building shown in Figure 14. Most of the work dealt with a typical interior bay, ADCB, the slab being symmetrical and continuous across boundaries AB and CD. The object was to find the slab stiffness for simultaneous equal rotation of all shear walls. It was assumed that as the walls rotated, their sections at the slab remained plane. A finite difference analysis was made for several cases with varying values of C, W, and L (Fig. 14 (a)). The value of Poisson's ratio used in all analyses was 0.19. The work is of interest because analyses were made using values of the shear wall length W small enough to approximate longish columns in a simple flat plate building.

In an attempt to obtain verification of the analytical results, a small model was constructed. The slab was made from asbestos cement sheet, and the walls from heavy steel plate, so that the inplane strains in the walls would be small enough to have only a negligible effect on the measured slab stiffness. Direct verification of the analytical results would have required a model with provision for restraining the slope across the continuous boundaries. Because this was difficult to achieve, the model tested had all edges free, and an analysis was made for this condition using the finite difference technique. The analytical and experimental results showed reasonable agreement, and it was accordingly assumed that the same method of analysis would give reliable solutions for the 'real' structure with continuous edges.

Analytical Results

Stiffness

The slab stiffness was defined by the parameter $k = M/D\theta$. For the smallest shear wall considered, $W = 0.15X$, k was found to vary with the shape of panel as shown in figure 15. For a square panel $k = 4.4$ approximately for zero overhang.

Edge panels have a stiffness less than half that of interior panels because the longitudinal edges are free except where connected to the shear walls. Three cases of edge panels were analysed, as a result of which the authors concluded that it would be sufficiently accurate to take the stiffness of an edge bay as 42% of the stiffness of an interior bay.

Effective Width

If an 'equivalent beam' be defined with stiffness EI, where $I = Y_e t^3/12$, Y_e being an effective width, such that the stiffness of the equivalent beam is the same as the slab stiffness, it can be shown that:

$$\frac{Y_e}{Y} = \frac{k\left(\frac{L}{L + W}\right)^2}{6(1-m^2)\left(\frac{Y}{L}\right)} \qquad (Y = \text{bay width})$$

The value of the slab stiffness parameter k was found from the finite difference analysis. The variation of the effective width ratio Y_e/Y with shape of panel is shown in Figure 15, for the smallest shear wall size W = 0.15X. It can be seen that the effective width for stiffness purposes was found to be considerably smaller than the panel width. For the common case of a square panel, the effective width was only 1/3 of the bay width; even for the rather unusual case of span: width = 4:1, the effective width was still only 80% of the bay width.

5.6 Aalami (1972)

Aalami considered the moment-rotation characteristics of square plates supported on central square columns, and postulated that the rotational stiffness must lie between two limits: in the upper limit the column-plate junction rotated as a rigid body; in the lower limit the column was assumed to add no bending stiffness to the plate, so that the deformation of the column-plate interface was a continuation of the deformation surface of the plate around the column. Although the upper limit would appear to correspond much more closely to the real situation, results for the lower limit only were presented.

Three ratios of column side, c, to plate size, L, were considered: 0.05, 0.10 and 0.15. Each was analysed by a finite difference procedure for three assumed boundary conditions of the plate, shown in Figure 16. The resulting stiffness coefficients M/Dθ are tabulated below. For comparison the stiffness coefficients calculated by Carpenter (1965) for rotation of an interior square column joint (assumed rigid) in a continuous square bay plate structure are also tabulated.

	Case 1 Simply Supported	Case 2 Fixed	Case 3 Sway Condition	Carpenter Continuous
0.05	4.17	4.25	4.10	4.7
0.10	5.53	6.24	5.40	6.45
0.15	6.78	6.59	6.59	

As would be expected, the results for Case 2, with all edges assumed fixed, give the best agreement with Carpenter's figures for a continuous plate structure.

Aalami drew two conclusions from these figures, for c/L ratios up to 0.15:

(i) that the stiffness was not sensitive to conditions at the boundaries;

(ii) that the stiffness was not significantly affected by the span of the plate in either direction, but depended predominantly on the column size alone.

Conclusion (i) is reasonable, although the boundary conditions assumed did affect the stiffness values by up to 20%.

Conclusion (ii) is not valid, and could not in any case have followed from the work presented in the paper since the same plate span was used in all analyses. Other work indicates what would be expected à priori, that the stiffness is a function of slab span as well as column size.

5.7 Summary of Stiffness Properties

5.7.1 Introduction

Evaluation of the published analytical and experimental data on plate stiffness is complicated by the fact that stiffness has been defined in different ways by different investigators, that some investigators have worked with circular, others with square, columns, and that they have used materials with different values of Poisson's Ratio.

In order to enable comparisons to be made, the published results will be adjusted, where necessary, to apply to square columns, and the effect of Poisson's ratio will be eliminated, thus:

Shape of Column

Where circular columns have been used, the side dimension "c" for square columns of equal area will be substituted; c = 0.885x diameter of circular column.

Poisson's Ratio (μ)

It will be assumed that the stiffness of the slab panels is proportional to the 'plate stiffness' D

$$D = \frac{E.t^3}{12(1-\mu^2)}$$

The panel stiffnesses will be discussed in terms of the dimensionless parameter $M/D\theta$, where M is the total moment applied to the slab at a column, and θ is the resulting slope at the column. With the assumption above, $M/D\theta$ is independent of μ.

All the work summarised here deals with the stiffness of interior panels of regular multi-panel flat plate floors. No information is available on the stiffness of edge or corner panels, apart from the limited work of Qadeer and Smith, who analysed a central panel in an infinitely long building one panel wide (Section 5.5).

5.7.2 Square Interior Panels

1. Stiffness defined in terms of the rotation of one interior column, while all other columns are held against rotation.
   This definition was used by Patel (1957) and Carpenter (1965).
   Results from their model tests, and from Carpenter's analytical work, are plotted in Figure 17.

The figure indicates the trend of increasing panel stiffness with increase in c/L ratio. However it is clear that more work is needed to verify and extend the range of the stiffness values.

Carry-over Factors

Patel and Carpenter agreed that the only column which received a significant carry-over moment was the first longitudinal fixed column in the direction of the applied moment. The actual value of the carry-over factor for moment to this column is uncertain. Carpenter's results suggested that the factor was about ¼ for a c/L ratio of 0.10, and increased with increasing c/L.

Carpenter also stated that carry-over factors to the other fixed columns had been found, in calculations not presented, to be about 0.035 to the diagonal column, and -0.045 to the transverse column.

**2. Stiffness defined in terms of the rotation of an entire transverse row of columns, adjacent rows being held fixed against rotation.

Brotchie defined stiffness in this manner, and Carpenter adapted his analysis to this definition for comparison with Brotchie's work.

The results of the two analyses are compared in Figure 18. Within the range of c/L common to both there is some divergence between the trends of the two graphs. No experimental check is available for this case.

**3. Stiffness defined in terms of equal rotation of every column.

This was the case investigated by Copley.

For square panels of side L, and a column side dimension c in the direction of the applied rotation, of 0.10L, Copley's analysis gave the following values for interior panel stiffness:

For column width = zero, $M/D\theta = 5.52$

For column width = 0.05L, $M/D\theta = 6.05$

It may be surmised that for a square column, width equal to 0.10L, the stiffness would have been about

$$M/D\theta = 6.6$$

This compares with a value of 6.5 for stiffness definition 1 (see Figure 17), and 5.5 for stiffness definition 2 (see Figure 18).

5.7.3 Rectangular Interior Panels

Copley's analysis indicated that panel stiffness, for all columns rotated, remained practically constant for any value of panel width B greater than the panel span L in the direction of the applied rotations.

For values of B/L greater than 0.8, the effective slab width for stiffness in an equivalent frame analysis, was a function of the span rather than the width of the panel (see Figure 13).

6. CONCLUSION

6.1 Vertical Loading

In the history of the development of flat slab design for uniform vertical loading, four major milestones are discernible:

1) The derivation by Nichols in 1914 of an expression for the total static moment in an interior panel;
2) The determination by Westergaard and Slater in 1921 of the distribution of moments within a panel;
3) The devising of an equivalent frame design method by Dewell and Hammill in the early 1930's;
4) The extensive programme of experimental and analytical work commenced at the University of Illinois in 1956, leading to the improved and rationalised design methods incorporated in the 1971 A.C.I. Code.

Although new analytical techniques for the determination of slab bending moments have been devised since Westergaard and Slater published the first solution in 1921, none has been useful as a design tool. Very few solutions for particular cases have been published and these have almost all been only for the simplest case of an interior panel in a uniformly loaded structure.

The design methods incorporated in current Codes are adequate for regular structures under uniform load, such as the 3 x 3 bay structures studied in the University of Illinois programme. Their adequacy is less certain for partial loading, and their applicability to cases of line loads or concentrated loads has not been explored. Their usefulness for structures in which, due to irregular layout or uneven loading, column-slab joint rotations are significant, is very doubtful.

6.2 Lateral Loading

The design of flat slab structures for lateral loads to-day is shrouded in ignorance as complete as that which marked design for vertical loads in C.A.P. Turner's day. Usable information on stiffness properties is virtually restricted to the limited data obtained for square interior panels by Carpenter (1965) and Patel (1957) shown in Figure 17. No reliable data are available on the stiffness of edge or corner panels, or on the moments carried over to adjacent columns. Clearly much work is needed before a rational method of design for lateral loading can be devised.

ACKNOWLEDGEMENT

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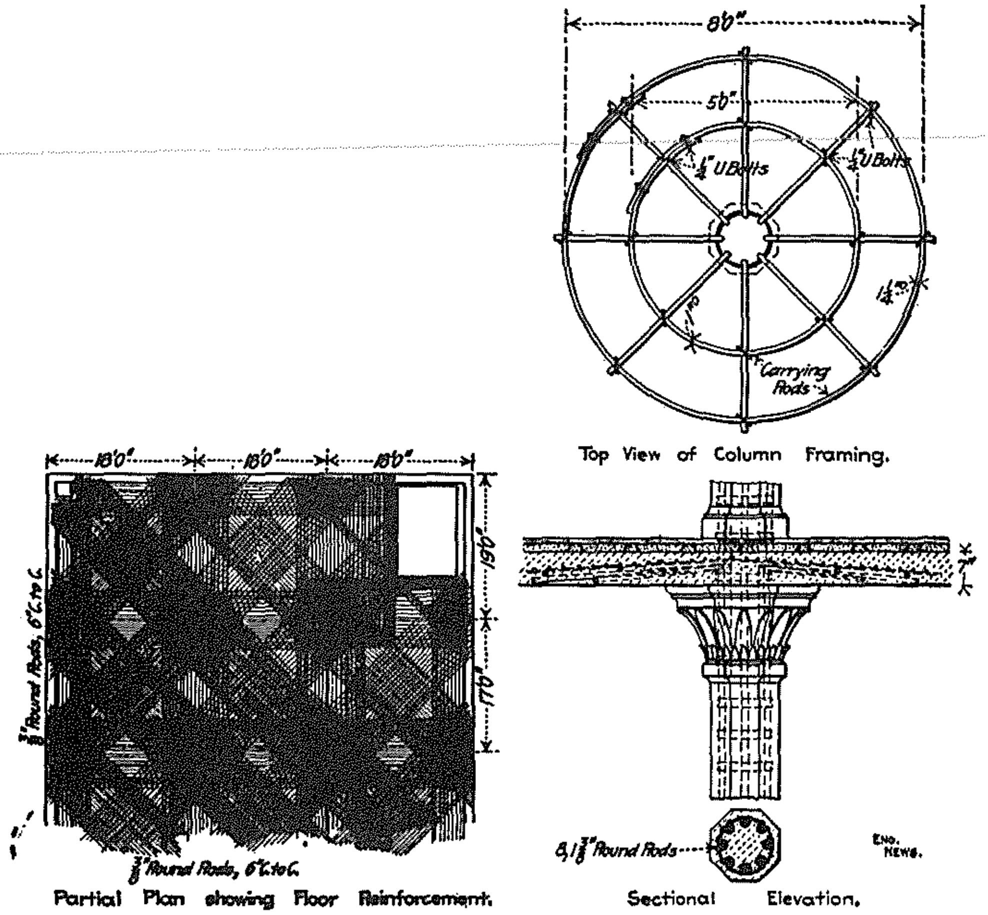
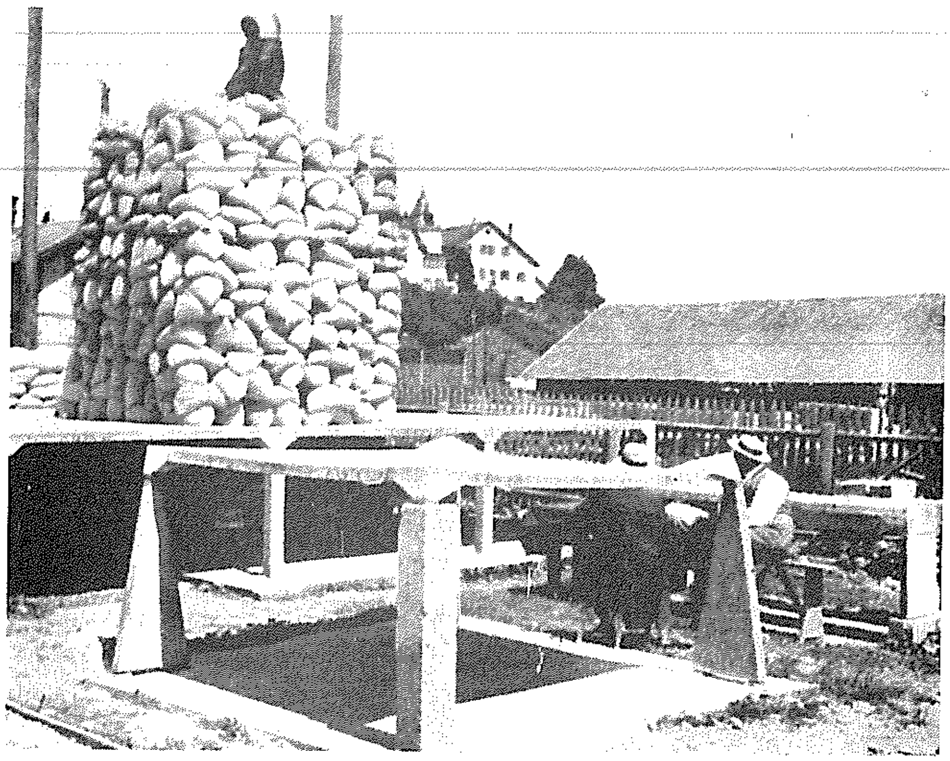


FIG. 3. "MUSHROOM" SYSTEM OF CONCRETE REINFORCEMENT PROPOSED BY C. A. P. TURNER.

EARLIEST KNOWN FLAT SLAB DRAWINGS

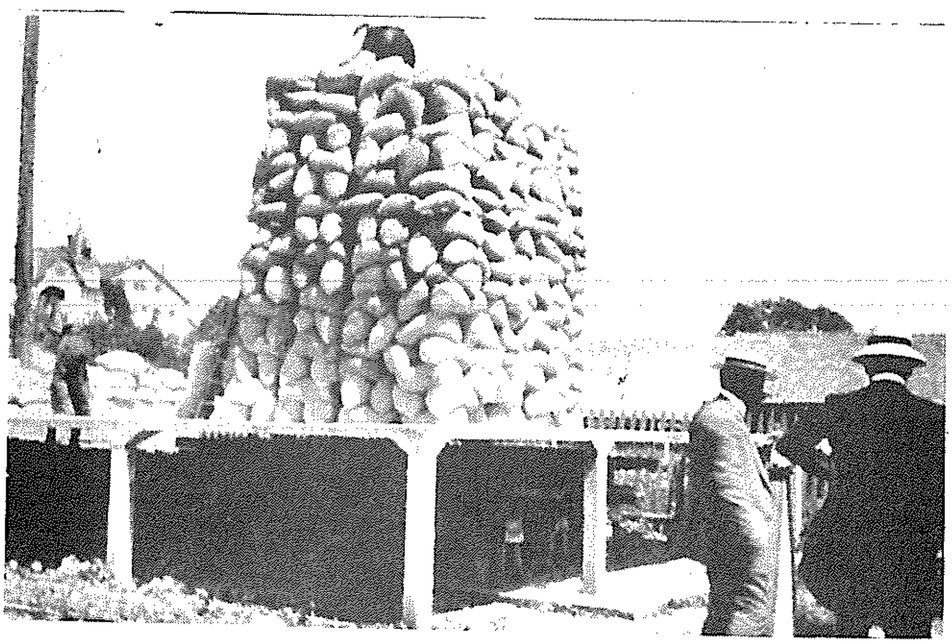
ENGINEERING NEWS, OCTOBER 12, 1905.

<u>Figure 1.</u>



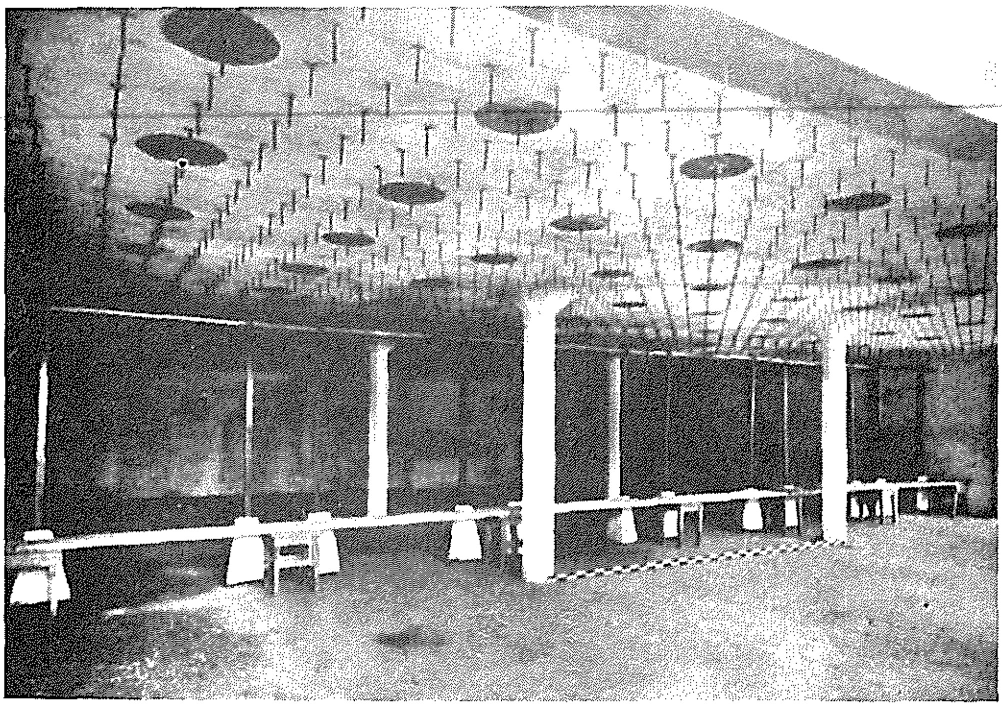
MAILLART TEST SLABS 1908

Figure 2.

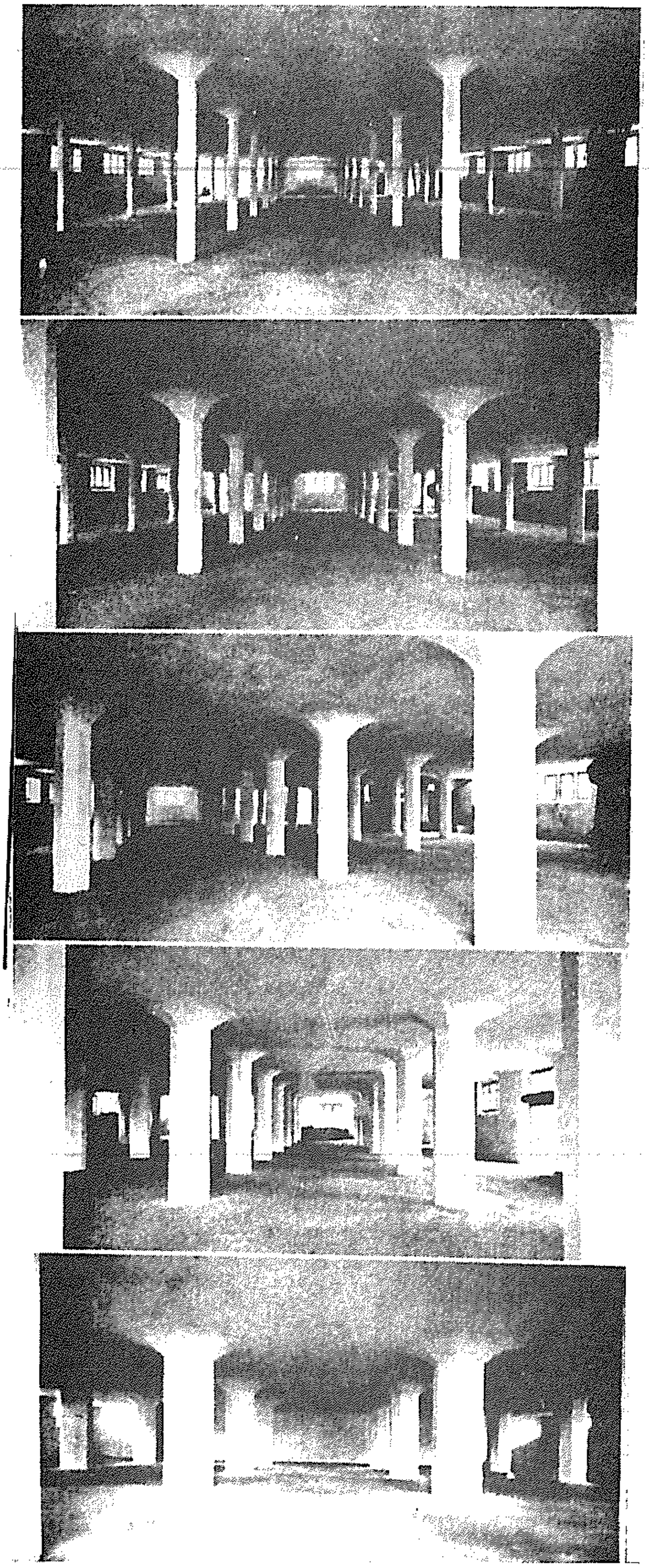


MAILLART NINE-BAY TEST FLOOR 1908

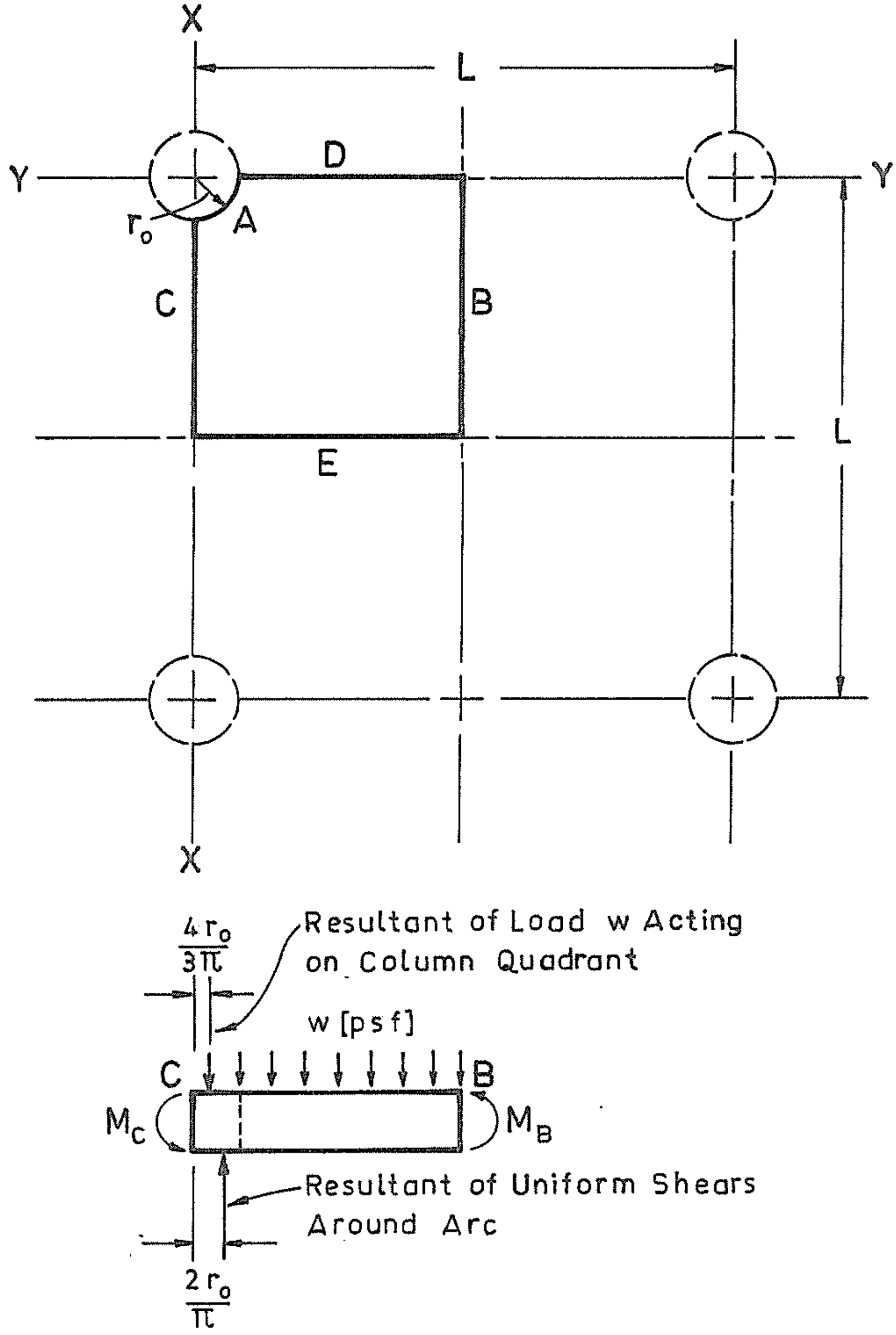
Figure 3.



MAILLART EXPERIMENTAL MUSHROOM-
SLAB BUILDING 1910.
Figure. 4.



FLAT SLAB BUILDING
FOR FEDERAL GRAIN
STOREHOUSE, ALTDORF,
SWITZERLAND, 1912,
DESIGNED AND BUILT
BY ROBERT MAILLART
<u>Figure 5.</u>



NICHOLS FREE BODY DIAGRAM
(From Nichols 1914)

Figure 6.

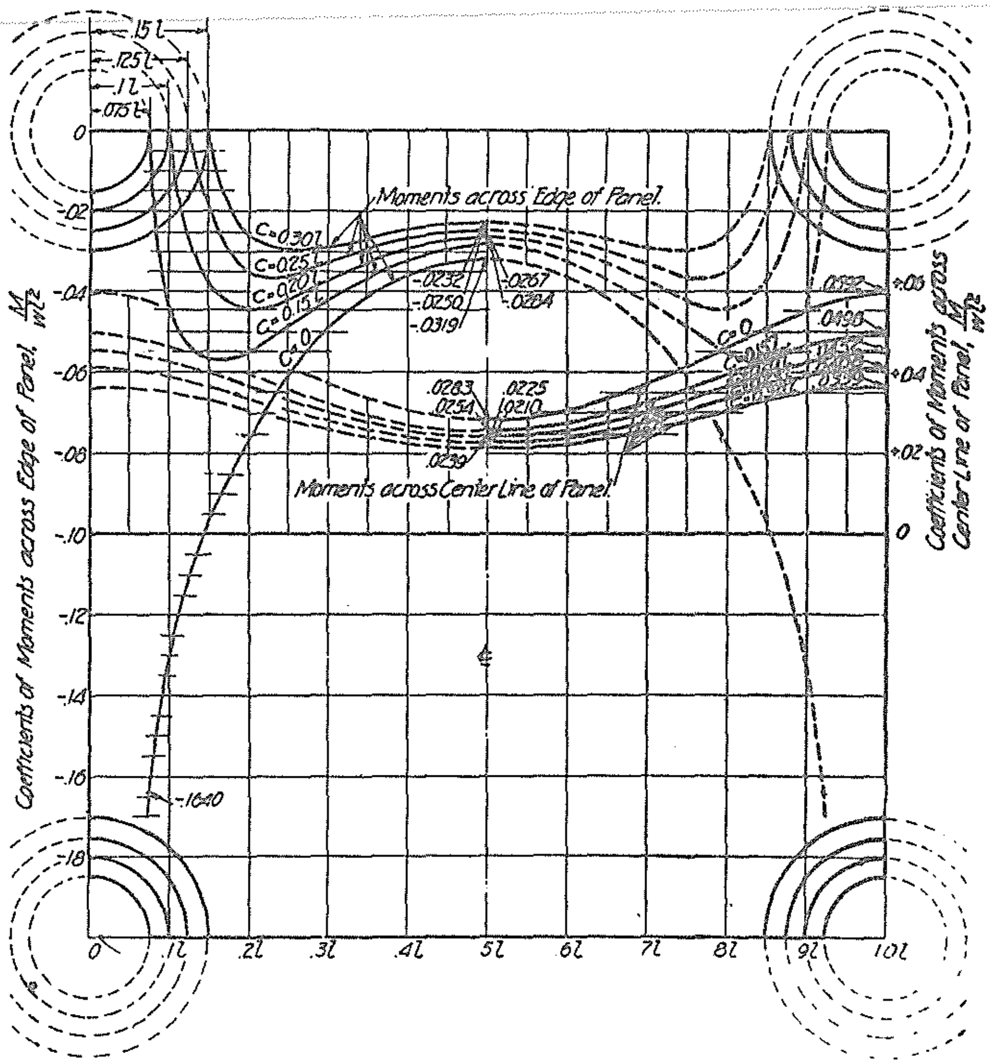


FIG. 14. COEFFICIENTS OF BENDING MOMENTS PER UNIT WIDTH IN A SQUARE INTERIOR PANEL OF A UNIFORMLY LOADED FLAT SLAB WHEN POISSON'S RATIO IS ZERO; MOMENTS ACROSS THE EDGE AND THE CENTER LINE.

BENDING MOMENT DISTRIBUTIONS

FROM WESTERGAARD & SLATER

Figure 7

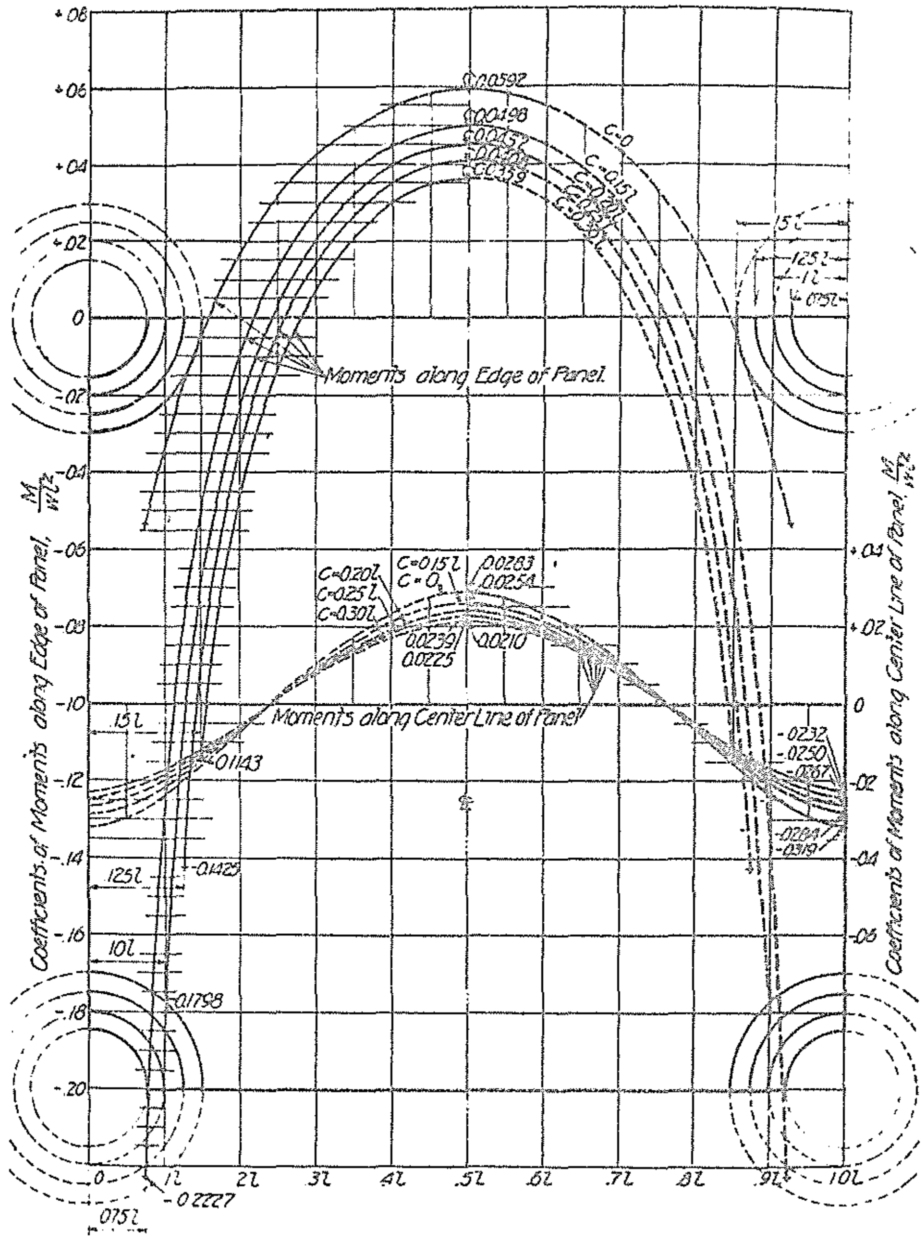
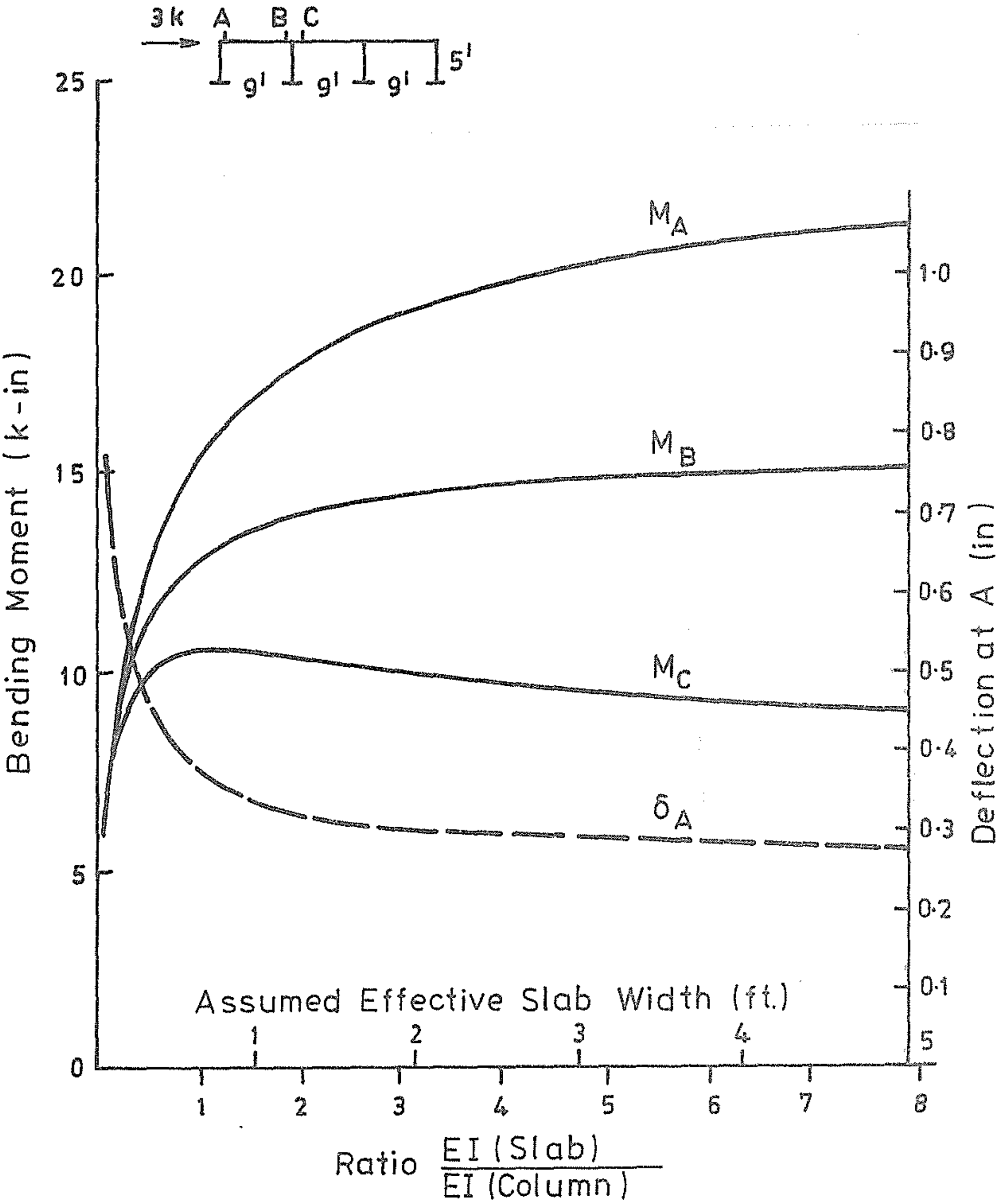


FIG. 15.—COEFFICIENTS OF BENDING MOMENTS PER UNIT WIDTH IN A SQUARE INTERIOR PANEL OF A UNIFORMLY LOADED FLAT SLAB WHEN POISSON'S RATIO IS ZERO; MOMENTS ALONG THE EDGE AND THE CENTER LINE.

BENDING MOMENT DISTRIBUTIONS

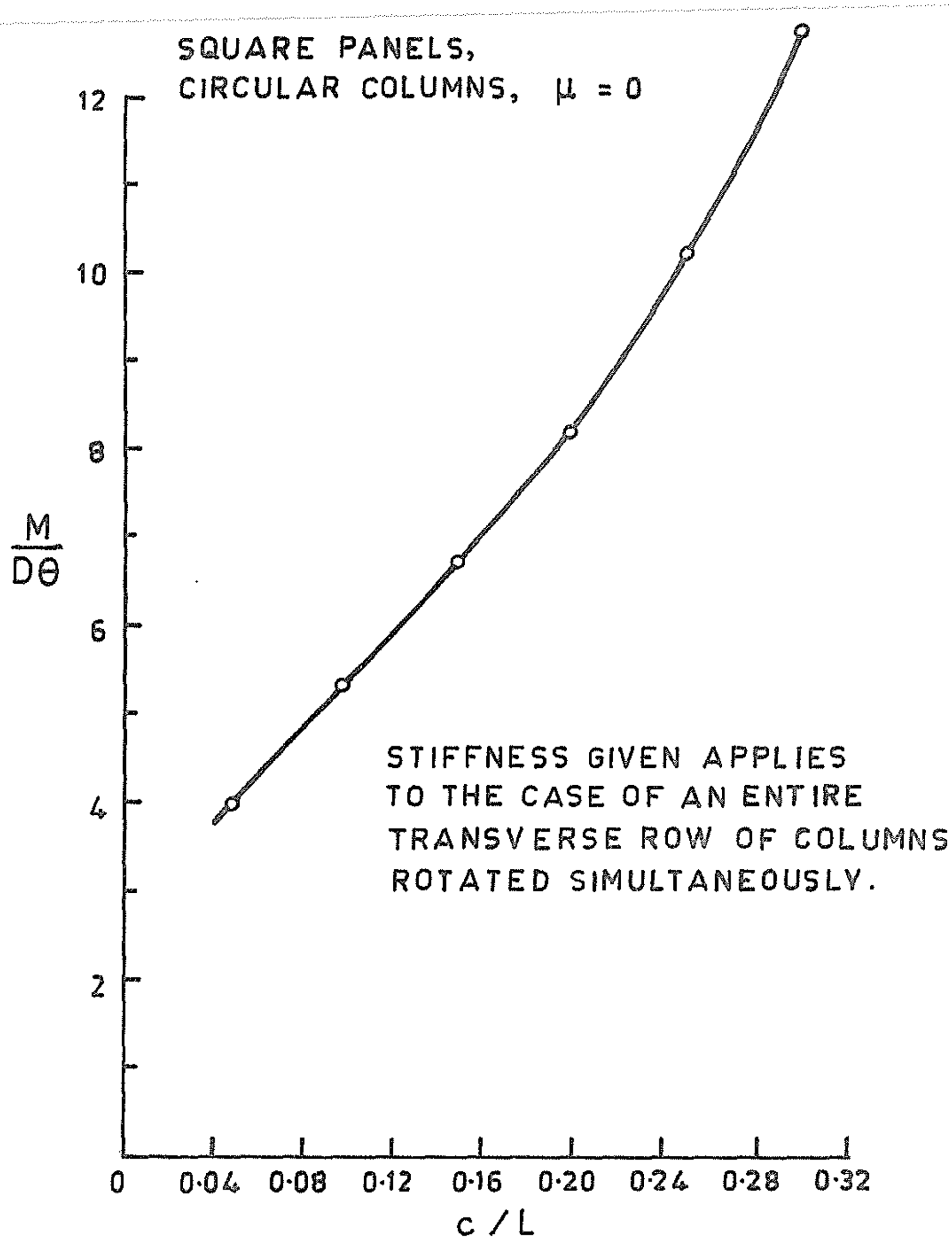
FROM WESTERGAARD & SLATER

Figure 8



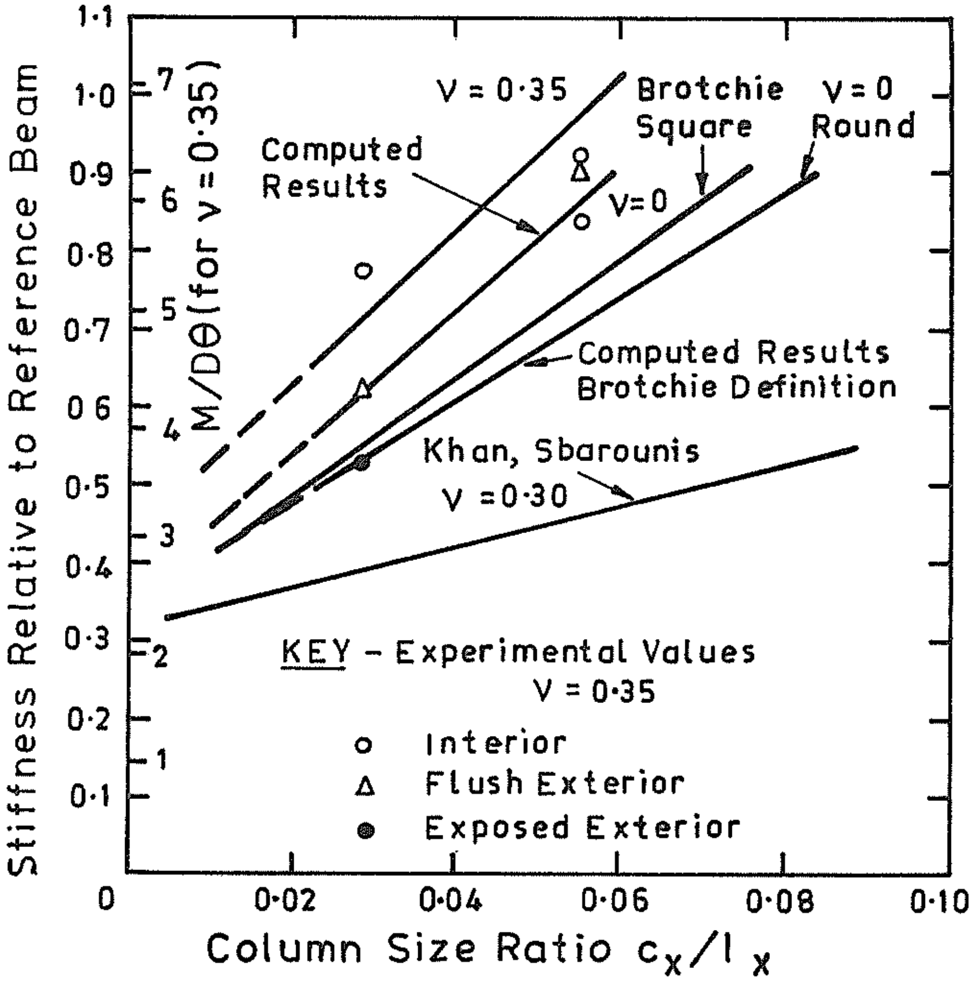
EQUIVALENT FRAME ANALYSIS
OF BERESFORD TEST FRAME.

Figure 9.



BROTCHIE AND RUSSELL STIFFNESS ANALYSIS

Figure 10.

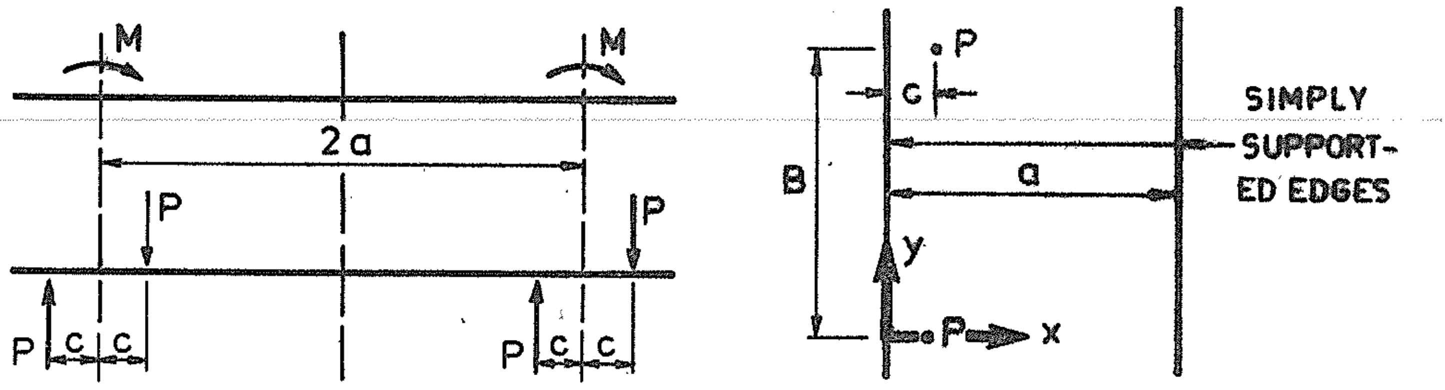


STIFFNESS OF SLAB ELEMENT

ν = Poisson's Ratio $c_x = \frac{1}{2}$ Column Dimension

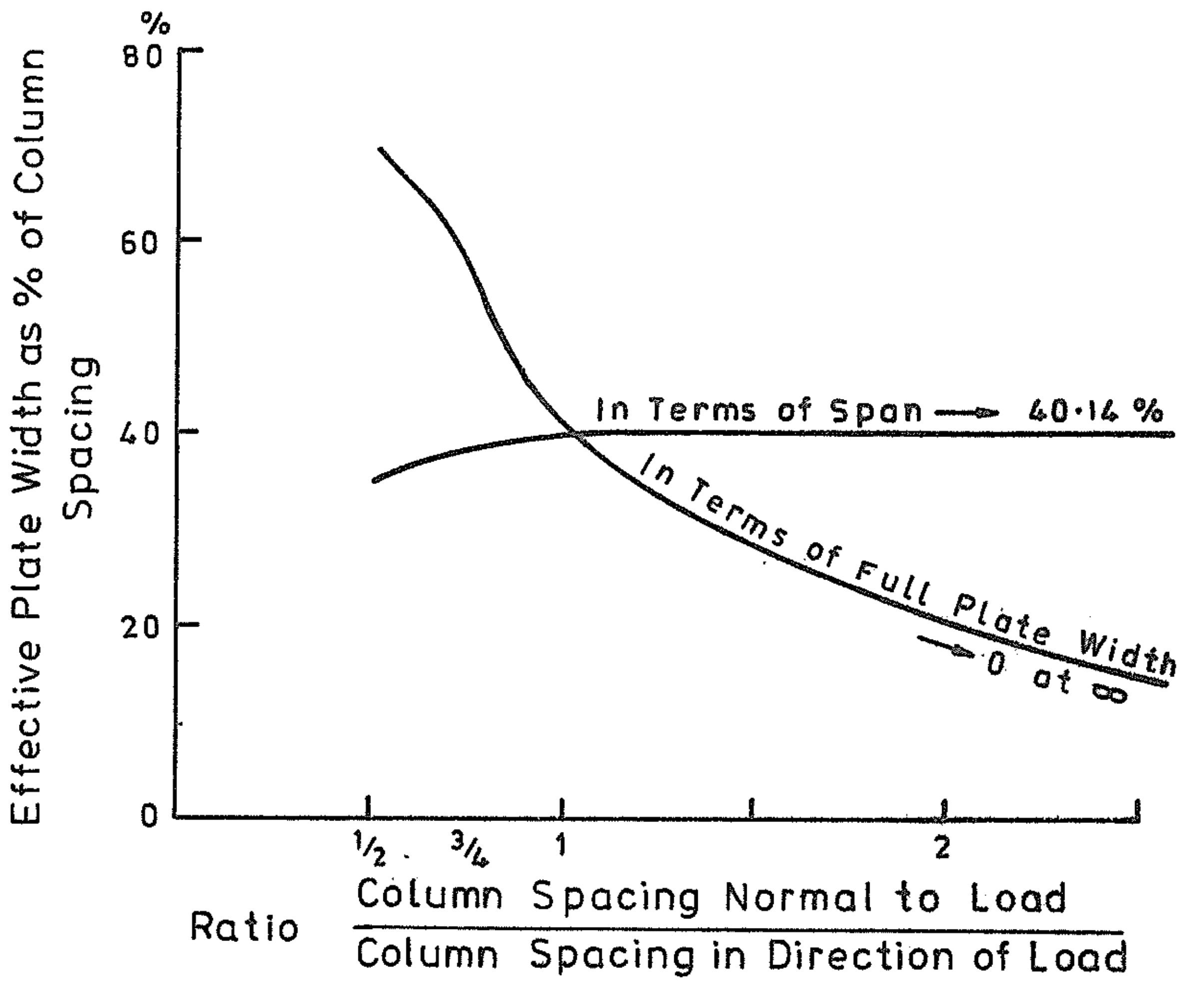
(From Carpenter (1965)

Figure 11.



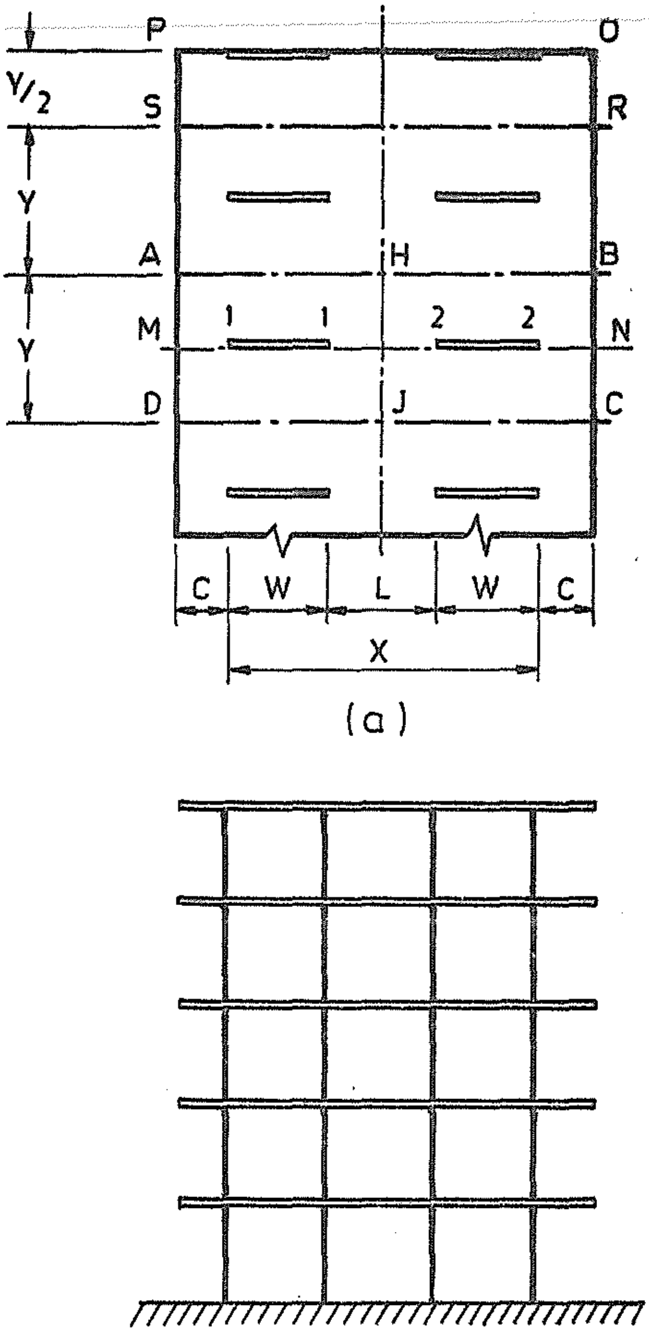
COPLEY ANALYTICAL MODEL

Figure 12.



COPLEY EFFECTIVE WIDTH ANALYSIS

Figure 13.

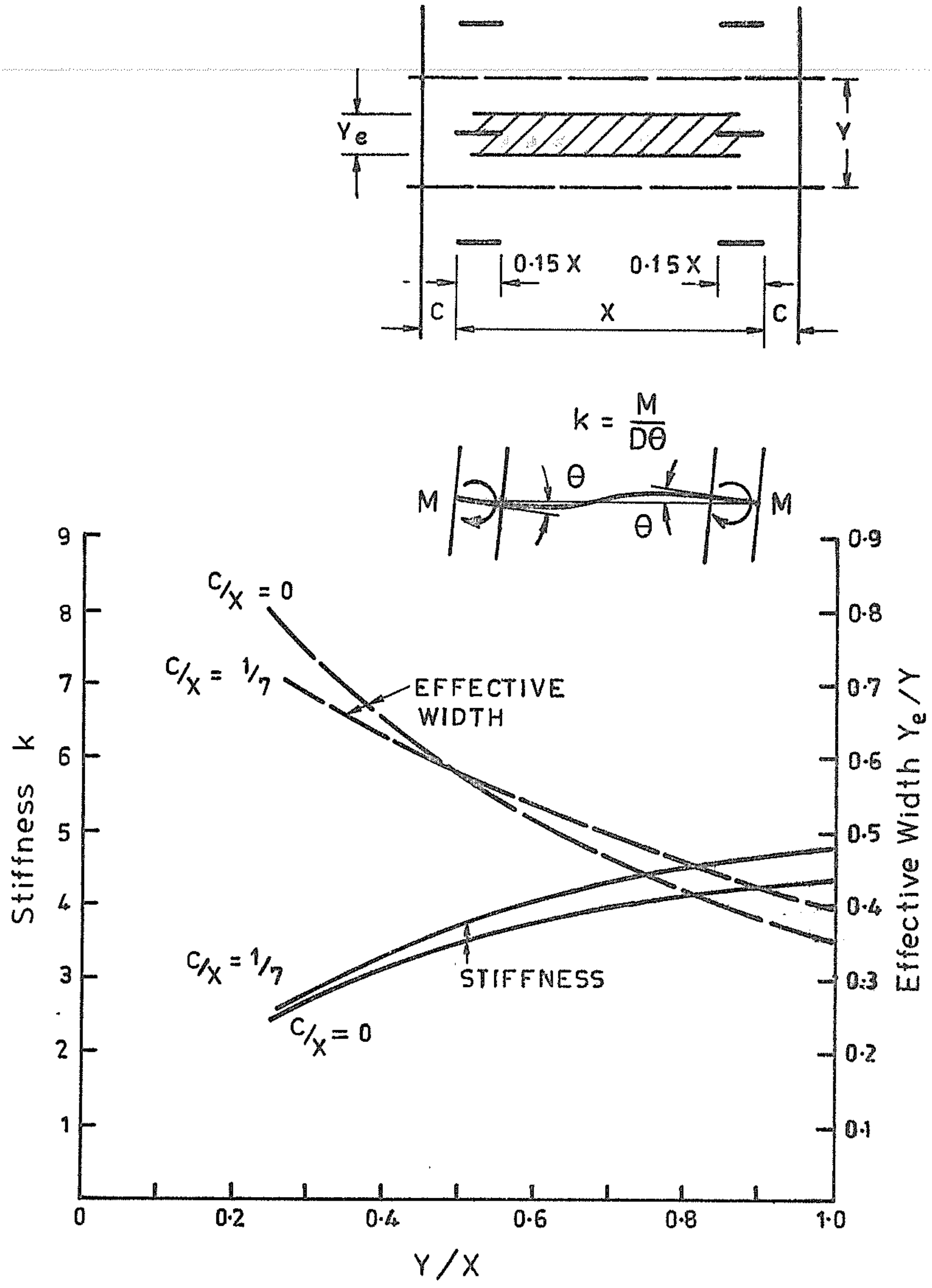


(b)

Slab and Cross-Wall Structure;
a) Plan: and b) Elevation.

STRUCTURE ANALYSED BY QADEER & SMITH (1969)

Figure 14.



QADEER AND SMITH ANALYSIS

Figure 15.

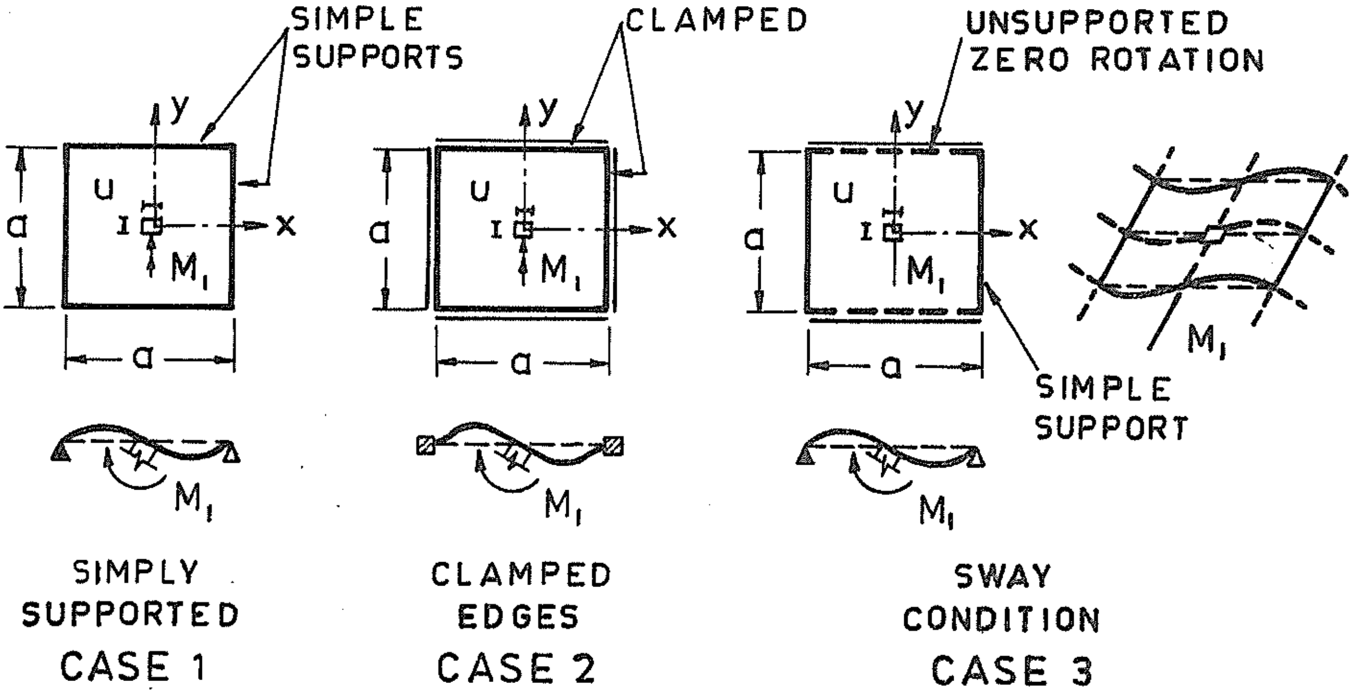
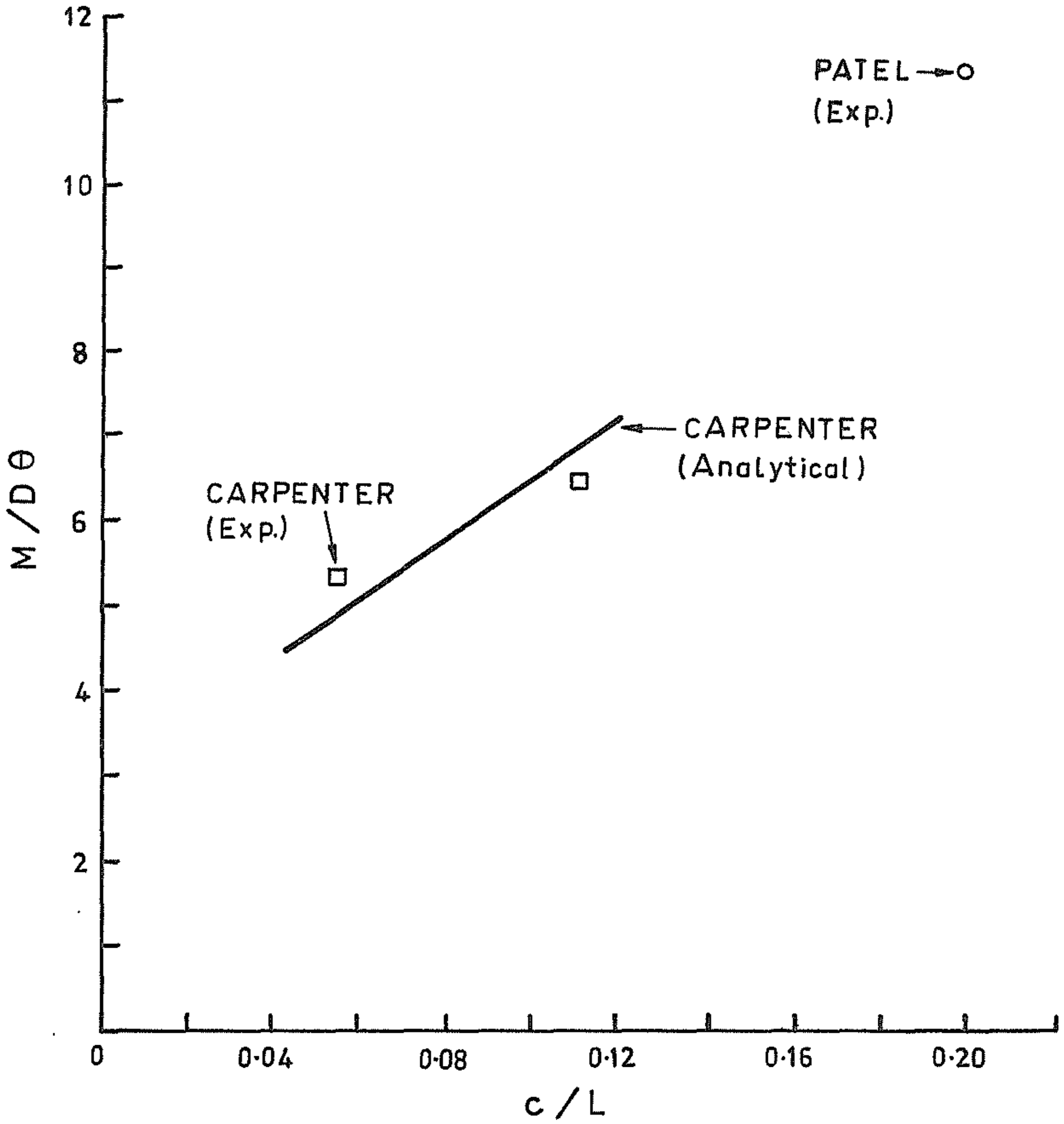


Fig. 2. Square Plates on Central Columns; Dimensions and Boundary Conditions of Three Cases Analysed.

BOUNDARY CONDITIONS USED IN ANALYSES BY AALAMI (1972)

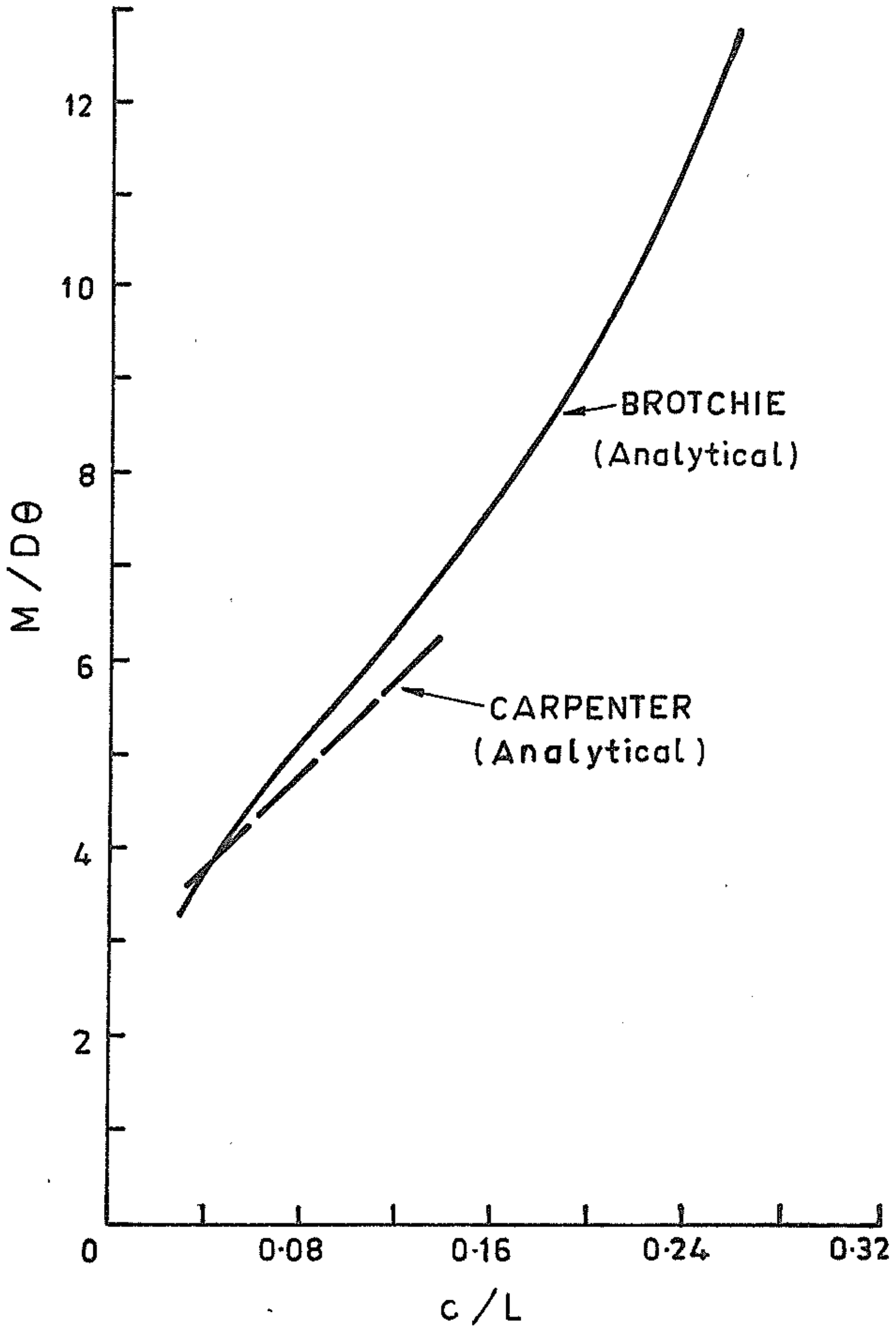
Figure 16.



Note: M is the Moment Applied to and Θ the Resulting Slope at an Interior Column Joint when all other Joints are Held against Rotation.

STIFFNESS OF INTERIOR SQUARE PANELS IN REGULAR FLAT SLAB FLOORS SUPPORTED ON SQUARE COLUMNS

Figure 17.



Note: M is the Moment Applied to and Θ the Resulting Rotation at each Interior Joint in an Entire Transverse Row, Other Joints being Held Fixed.

"BROTCHIE STIFFNESS" OF INTERIOR SQUARE PANELS IN REGULAR FLAT SLABS SUPPORTED ON SQUARE COLUMNS.

Figure 18.