AN EXPERIMENTAL STUDY OF TENSION

STIFFENING IN REINFORCED CONCRETE TENSION

MEMBERS UNDER SHORT-TERM AND

LONG-TERM SERVICE LOADS

by

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An experimental investigation of the behaviour of reinforced concrete in tension and the mechanisms of tension stiffening in reinforced concrete are presented. Six reinforced concrete specimens were constructed and tested in axial tension. Each specimen consisted of a 100 mm by 100 mm by 1100 mm concrete prism reinforced with a single reinforcing bar placed parallel to the long dimension of the prism and located at the centroid of each 100 mm by 100 mm cross-section. Each specimen was moist cured for a period prior to testing. Four specimens were tested to failure under the application of a monotonic axial tensile load applied to the ends of the reinforcing bar protruding from the concrete prism. Two of these specimens were tested immediately after moist curing before significant drying shrinkage had commenced and two specimens were allowed to shrink for two weeks prior to the commencement of loading. A further two specimens were subjected to sustained axial tension for a period of 2 months, with shrinkage commencing at the age of first loading. The strain distribution along the reinforcing bar was monitored using strain gauges, and the strain readings were used to calculate the variation of tensile force carried by the reinforcement and hence the variation of concrete tensile force along the specimen. The location, spacing and width of cracks in the short-term tests were monitored as the load was increased and, in the sustained load tests, the development of cracking with time was monitored. LVDTs were used to measure the total elongation of the specimen throughout the short-term tests and Demec gauges were used in the long-term tests. Material properties, including the creep and shrinkage characteristics of the concrete, were measured on companion specimens.
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ABSTRACT
An experimental investigation of the behaviour of reinforced concrete in tension and the mechanisms of tension stiffening in reinforced concrete are presented. Six reinforced concrete specimens were constructed and tested in axial tension. Each specimen consisted of a 100mm by 100 mm by 1100 mm concrete prism reinforced with a single reinforcing bar placed parallel to the long dimension of the prism and located at the centroid of each 100mm by 100 mm cross-section. Each specimen was moist cured for a period prior to testing. Four specimens were tested to failure under the application of a monotonic axial tensile load applied to the ends of the reinforcing bar protruding from the concrete prism. Two of these specimens were tested immediately after moist curing before significant drying shrinkage had commenced and two specimens were allowed to shrink for two weeks prior to the commencement of loading. A further two specimens were subjected to sustained axial tension for a period of 2 months, with shrinkage commencing at the age of first loading. The strain distribution along the reinforcing bar was monitored using strain gauges, and the strain readings were used to calculate the variation of tensile force carried by the reinforcement and hence the variation of concrete tensile force along the specimen. The location, spacing and width of cracks in the short-term tests were monitored as the load was increased and, in the sustained load tests, the development of cracking with time was monitored. LVDTs were used to measure the total elongation of the specimen throughout the short-term tests and Democ gauges were used in the long-term tests. Material properties, including the creep and shrinkage characteristics of the concrete, were measured on companion specimens.

KEY WORDS
Axial tension; cracking; creep; deformation; laboratory experiments; reinforced concrete; serviceability; shrinkage; tension stiffening; time effects.
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1. INTRODUCTION

Tension stiffening in a reinforced concrete member arises from tensile stresses carried by the concrete. Tension stiffening contributes significantly to the stiffness of the member and is an important consideration when designing for deflection and crack control at the serviceability limit states. Tension stiffening is particularly significant in relatively lightly reinforced members, where the actual stiffness may be several times larger than the stiffness calculated on the basis of fully-cracked cross-sections, where the tensile concrete is ignored and only the embedded tensile reinforcement is considered. Tension stiffening increases with an increase in tensile stress in the concrete. Conversely, tension stiffening decreases when the tensile stress in the concrete drops and, under constant load, this is caused either by cracking, by tensile creep or by a time-dependent deterioration of bond. Cracking can be caused by external loads or by restraint to imposed deformations, such as drying shrinkage.

Consider the uniaxially loaded tension member shown in Figure 1a. Before cracking, the concrete tensile stress increases with load. When the stress in the concrete first reaches the tensile strength at a particular section, cracking occurs at the applied load $P_{cr}$ (see Figure 1b). First cracking occurs at the weakest cross-section and this is usually assumed to occur when the concrete tensile stress reaches the lower characteristic value of the direct tensile strength, $f'_{ct}$. When cracking first occurs, the stress in the concrete at the crack drops to zero. The concrete stress increases with distance from the crack due to the steel-concrete bond, until at some distance $s$ from the crack, the concrete stress is no longer affected by the crack, as shown in Figure 1c. Slip at the concrete-steel interface in the region of significant bond stress ($s$ on either side of the crack) causes the crack to open.

A relatively small increase in load will cause a second crack to develop at a cross-section at some distance $x \geq s$ from the first crack, thereby reducing the concrete stress in the vicinity of that crack. Eventually, under increasing load, primary cracks form at somewhat regular intervals along the member and the primary crack pattern is established. The concrete tensile stress at each crack is zero, rising to a maximum value $\sigma_c$ (less than the tensile strength of the concrete) mid-way between adjacent cracks, as shown in Figure 1d.

Cracking is therefore accompanied by a drop in the average tensile stress carried by the concrete and, hence, a reduction in tension stiffening. After the primary crack pattern is established, further increases in load may result in further slip at the concrete-steel interface causing cover-controlled cracks to develop between the primary cracks and a gradual breaking down of the bond between the steel and the concrete, thereby reducing tension stiffening still further, as shown in Figure 1b. Under sustained load, $\sigma_c$ may gradually reduce, primarily due to cracking and bond breakdown caused by drying shrinkage and, to a lesser extent, due to tensile creep. The time-dependent change in tension stiffening is not well understood and has received relatively little research attention.
Figure 1: Tension stiffening in an axially loaded tension member.

Just before first cracking when the axial force just reaches $P_{cr}$, the average stress in the concrete equals the tensile strength of the weakest cross-section. The average stress in the reinforcement just before cracking depends on the amount of shrinkage that has taken place in the member prior to cracking (and is in fact compressive for usual levels of early shrinkage). The amount of shrinkage before cracking greatly affects the cracking load, $P_{cr}$.

Figure 2b shows the idealized instantaneous and time-dependent responses of the concentrically reinforced concrete tension member shown in Figure 2a, both before and after cracking. As described earlier, the instantaneous response (curve OAB in Figure 2b) is linear up until first cracking at $P = P_{cr}$ and non-linear after cracking. Before cracking, the instantaneous tension stiffening strain $\Delta \varepsilon_{int}$ (which is the difference between the strain in the specimen and strain in the bare bar) increases with load, but after cracking, $\Delta \varepsilon_{int}$ decreases as $P$ increases, as shown.

If the load $P$ is held constant with time, the average axial strain is affected by creep and shrinkage. In a member that does not shrink, tensile creep causes a softening of the concrete in tension and a relatively small change in the load-deformation response with time (as shown by the dashed curve labeled 'after creep only' in Figure 2b). Tensile creep of the concrete sheds some of the tensile force carried by the concrete into the bonded reinforcement, thereby reducing tension stiffening with time.

Shrinkage before the commencement of loading ($\varepsilon_{shrink}$) causes a significant change in the load-deformation response (which is shown in Figure 2b as curve O'A'D). Shrinkage causes the member to shorten before loading commences and the concrete strain before loading is represented by point O'. When loading commences, the load-deformation curve moves to
Figure 2: Instantaneous and time-dependent response of axially loaded tension member.

the left in Figure 2b, as shown by the dashed curve O'A'. Restraint to shrinkage causes a gradual build-up of tension in the concrete and this reduces the cracking load from \( P_{cr} \) to \( P_{cr,sh} \), as shown.

At load \( P (> P_{cr}) \) in Figure 2b, the instantaneous tension stiffening strain \( \Delta \varepsilon_{eff} \) is represented by the horizontal distance BE and, under sustained loads, this reduces to CE due to creep of the concrete and reduces further to DE if creep is accompanied by further shrinkage. It is often assumed that tension stiffening reduces with time under sustained loads to about 50% of its instantaneous value, but this is yet to be conclusively demonstrated.

In this report, results are presented of an experimental study of tension stiffening in axially loaded prisms exposed to various periods of drying. The change in tension stiffening with time in cracked reinforced concrete elements subjected to sustained loads and drying shrinkage is also reported.

2. EXPERIMENTAL PROGRAM

2.1 Overview:

The experimental program involved the testing of several reinforced concrete prisms in axial tension. Each concrete prism was of square cross-section (100mm by 100mm) and was 1100 mm long and contained a single reinforcing bar running longitudinally through the centroid of each cross-section, as shown in Figure 3. The tensile axial load was applied to the ends of the reinforcing bar protruding from each end of the concrete prism.

Four of the specimens were tested under monotonically increasing deformation up to yielding of the reinforcing steel bar (the short-term tests). The other two specimens were
tested under a constant, sustained, service load for a period of about 50 days (the long-term tests). Companion specimens were also tested to measure the time-dependent creep and drying shrinkage characteristics of the concrete. In addition, the instantaneous compressive strength and the elastic modulus of concrete were measured on standard 150 mm diameter concrete cylinders, the indirect concrete tensile strength was measured on standard cylinders using the Brazil test and the flexural tensile strength was measured on 100mm by 100mm by 600mm concrete prisms.

For the short-term tests, the development of cracking, the crack location and crack width, the average elongation of the specimen and the steel strains were recorded as the external load increased. For the long-term tests, the development of cracking, the crack location and crack width, and the average elongation of the specimen were recorded as time increased.

The objectives of the experimental program were

- To quantify tension stiffening in uniaxial tension specimens under increasing load and to measure the effects of concrete shrinkage prior to the application of the applied load;

- To investigate tension stiffening under sustained service loads and to analyse the effect of creep and shrinkage on the decay of tension stiffening with time, if any;

- To assess the effect of time-dependent cracking on tension stiffening; and

- To better understand the mechanisms of tension stiffening under in-service conditions and thereby to facilitate development of reliable numerical models of the in-service behaviour of reinforced concrete.

### 2.2 Specimen Layout and Test Parameters:

Details of the concrete prisms tested in this study are shown in Figure 3. Part of the tension force $P$ is transferred into the concrete from the reinforcement bar through bond at the steel-concrete interface. To investigate the influence of reinforcement ratio (i.e. the ratio of the cross-sectional areas of steel and concrete, $A_s/A_c$) on tension stiffening, two bar sizes were used, namely 12 mm and 16 mm diameter bars. Therefore, the reinforcement ratio for each specimen was either 1.11% or 2.04%. Over the middle 600 mm length of each specimen, 25 strain gauges were attached to the reinforcing bar at 25mm centres to monitor steel strains throughout the test, as indicated in Figure 3.

Four of the specimens (STN12, STN16, STS12 and STS16) were tested under short-term monotonically increasing deformation, with loading continuing into the post-yield range up to a 3% elongation. These four tests are referred to as the short-term tests. The other two specimens (LTN12A and LTN12B) were subjected to constant sustained service loads for a period of about 50 days and are referred to as the long-term tests.
Figure 3: Details of test specimens

The first two letters in the designation of each specimen indicate the test duration; "ST" for short-term and "LT" for long-term. The third letter indicates whether or not the specimen commenced drying and began to shrink prior to the application of load; "S" if yes (i.e., significant shrinkage had occurred before loading) and "N" if no (i.e., relatively little drying shrinkage had occurred prior to loading). The next two digits indicate the reinforcing bar diameter, either 12 mm or 16 mm. For the two long-term tests, the final letter A and B distinguishes the two different load cases.

The short-term tests were conducted in an INSTRON universal testing machine under displacement control. As the deformation gradually increased, the applied load $P$, the elongation of the specimen, the location of cracking and the variation of strain along the steel were recorded. The elongation of the specimen was monitored by LVDTs on two opposite sides of the concrete prisms over an 890 mm gauge length, as can be seen in Figure 4.

For the long-term tests, the loading frame shown in Figure 5 was used to apply the constant sustained axial tension. Loads were imposed on each specimen by an adjustable anchor-support at the top of the rig and monitored using a load cell at the base of the rig. Five Demec targets were attached on opposite sides of each concrete prism at 250 mm centres in order to measure the average elongation of the specimen.

Loads, LVDT readings and steel strains were recorded electronically using an HBM amplifier and crack locations, crack widths and Demec readings were recorded manually. Crack widths were measured using a microscope with a magnification factor of 40.
Figure 4: Short-term test set-up and LVDT.

Figure 5: Long-term testing rig.
2.3 Constructions of Specimens and Loading Details:

All the specimens were cast from the same batch of concrete and all were cured under wet burlap for a period of three weeks to facilitate strength gain and to delay the commencement of drying shrinkage. The Demec gauge targets were glued onto the concrete after removing the burlap. For the short-term tests, specimens STN12 and STN16 were tested immediately after wet curing, so that little if any shrinkage had occurred at the time of testing. Specimens STS12 and STS16 were uncovered and drying shrinkage commenced three weeks before the beginning of testing.

Specimen LTN12A was subjected to an axial tensile force of 40 kN (i.e. 350 MPa in the bare steel bar) for 50 days, while Specimen LTN12B was subjected to an axial tensile force of 20 kN (i.e. 177 MPa in the bare steel bar). The 20 kN axial force was then maintained for the remainder of the testing period. Each of the long-term test specimens was wet cured until the tests started, so that shrinkage effectively commenced at the beginning of the sustained load period.

Companion concrete cylinders, prisms and shrinkage samples were cast concurrently with the test specimens. At the time of each specimen test, indirect tensile strength, flexural tensile strength, compressive strength and elastic modulus of concrete were measured.

Shrinkage prisms, each 600mm long with 100 mm by 100 mm cross section, were used to measure the development of concrete shrinkage with time. Brass plugs at 250 mm centres were embedded in opposite surfaces of the prism before casting (as shown in Figure 6) and a Demec gauge was used to measure the average strain between two plugs in the prism (i.e. the shrinkage strain). The deformation between the brass plugs was measured from the second day after casting including during the period of moist curing.

Creep was measured in the standard creep rigs shown on the right hand side of Figure 6. There were three sets of shrinkage and creep measurements during the whole testing procedure according to the different concrete batches of the main specimens and the different ages of loading.

Figure 6: Shrinkage specimens and creep rigs.
3. MATERIAL PROPERTIES

3.1 Instantaneous Properties of Concrete:

The measured compressive strength $f'_c$, the indirect tensile strength $f'_{tt}$, the flexural tensile strength $f'_{cf}$ and the elastic modulus of concrete $E_c$ for each specimen at the time of first loading is given in Table 1.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$f'_c$ (MPa)</th>
<th>$f'_{tt}$ (MPa)</th>
<th>$f'_{cf}$ (MPa)</th>
<th>$E_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STN12</td>
<td>21.56</td>
<td>2.04</td>
<td>3.05</td>
<td>22400</td>
</tr>
<tr>
<td>STN16</td>
<td>21.56</td>
<td>2.04</td>
<td>3.05</td>
<td>22400</td>
</tr>
<tr>
<td>STS12</td>
<td>24.73</td>
<td>2.15</td>
<td>3.08</td>
<td>21600</td>
</tr>
<tr>
<td>STS16</td>
<td>24.73</td>
<td>2.15</td>
<td>3.08</td>
<td>21600</td>
</tr>
<tr>
<td>LTN12A</td>
<td>21.56</td>
<td>2.04</td>
<td>3.05</td>
<td>22400</td>
</tr>
<tr>
<td>LTN12B</td>
<td>21.56</td>
<td>2.04</td>
<td>3.05</td>
<td>22400</td>
</tr>
</tbody>
</table>

3.2 Creep of Concrete:

Creep was measured on companion cylinders loaded in a standard creep rig. Identical unloaded cylinders were kept under the same environmental condition to measure the shrinkage strain that occurred simultaneously with the creep strain in the loaded cylinders. The difference between the displacements of the loaded and unloaded samples is the mechanical displacement (i.e. the creep plus instantaneous displacement). The creep coefficient is the ratio of the creep strain to instantaneous strain at different concrete ages and typical measured creep coefficient versus time curves are given in Figure 5.

![Figure 7: Creep coefficient versus time](image-url)
3.3 Shrinkage:

In order to investigate the correct drying shrinkage strain and minimize the size effect on shrinkage measurement, the shrinkage prisms were designed with the same cross-section and shape as the testing specimens. The construction and measurement approaches are described in Section 1.3.

Figure 8 shows the typical development of shrinkage for concrete that initially began to dry at age 25 days.

![Shrinkage vs Concrete Age](image)

**Figure 8:** Drying shrinkage development

4. SHORT-TERM TEST RESULTS AND DISCUSSION

4.1 STN12:

Load versus Average Axial Strain:

Specimen STN12 was subjected to monotonically increasing deformation soon after the end of moist curing. The elastic modulus of concrete at the time of testing at age 32 days was $E_c = 22400$ MPa (and with $E_s = 200000$ MPa, $n = 8.93$) and the tensile strength of concrete was $f_t = 2.04$ MPa. Prior to loading, the measured drying shrinkage strain in the unreinforced companion member was small at $\varepsilon_{dl0} = -28 \times 10^{-6}$ and the calculated strain in the specimen (accounting for the restraint provided by the reinforcement – see Figure 2) was $-25 \times 10^{-6}$.

The average strain versus applied load measured using LVDTs throughout the tests is plotted in Figure 9, together with the bare bar response for the specimen. At any load level, the difference in strain between the specimen and the bare bar is the *tension stiffening strain* and it represents the contribution of the concrete to the member stiffness.
**Figure 9:** Average strain ($\varepsilon_{s,avg}$) versus load $P$ for STN12

Initially, prior to first cracking in portion O'B of the curve in Figure 9, the specimen is at its stiffest and the load-strain curve is steep. At first cracking (Point B when $P = P_{cr} = 21.5$ kN), there is an abrupt change of stiffness and the stiffness continues to degrade under increasing deformation as further cracks occur (portion BC). As the load increases, the tension stiffening strain gradually reduces. In total, 5 primary cracks occurred as loading progressed, as indicated by the numbered peaks in the curve in Figure 9.

Table 1 provides values of average axial strain and tension stiffening strain at selected values of total applied load $P$, together with the average force carried by the concrete and the steel, $P_{concrete}$ and $P_{steel}$, respectively, within the prism.

<table>
<thead>
<tr>
<th>Applied load, $P$ (kN)</th>
<th>0</th>
<th>10</th>
<th>21.1</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average strain, $\varepsilon_{s,avg}$ ($\times 10^6$)</td>
<td>-25</td>
<td>9.2</td>
<td>55.5</td>
<td>561</td>
<td>1148</td>
<td>1795</td>
<td>2355</td>
</tr>
<tr>
<td>Tension stiffening strain $\varepsilon_{s}$ ($\times 10^6$)</td>
<td>25</td>
<td>433</td>
<td>878</td>
<td>545</td>
<td>401</td>
<td>196</td>
<td>79</td>
</tr>
<tr>
<td>Avge force in concrete $P_{concrete}$ (kN)</td>
<td>0.57</td>
<td>9.79</td>
<td>19.8</td>
<td>12.3</td>
<td>9.1</td>
<td>4.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Avge force in steel bar $P_{steel}$ (kN)</td>
<td>-0.57</td>
<td>0.21</td>
<td>1.3</td>
<td>12.7</td>
<td>25.9</td>
<td>40.6</td>
<td>53.2</td>
</tr>
</tbody>
</table>

**Variation of Steel Strains:**

The variation of steel strains over a 600mm gauge length of the reinforcing bar were measured using strain gauges as the test progressed and the variation in force in the reinforcing steel was determined from the measured strains. The variation in concrete force was obtained by subtracting the force in the steel at each strain gauge location from the total force applied to the specimen.

Figure 10 show the variation in the tensile forces carried by steel and concrete at different loading stages for the short-term specimen STN12.
(a) Just before first cracking, $P = 21.1$ kN.

(b) Just after first cracking, $P = 17.8$ kN.

(c) Just before 2nd crack, $P = 22.4$ kN.

(d) Just after 2nd crack, $P = 17.4$ kN.

(e) Just before 3rd crack, $P = 21.8$ kN.

(f) Just after 3rd crack, $P = 19.6$ kN.

(g) At $P = 40.0$ kN.

(h) At $P = 50.0$ kN.

Figure 10: Variation of forces in steel and concrete at different stages of loading (STN12).
Crack Pattern and Crack Width:

The crack locations and crack numbers for STN12 are shown in Figure 11 and the crack widths at various stages of loading are given in Table 2, together with the steel stress (\(\sigma_s\)) at the crack at each loading stage.

![Figure 11: Crack numbers and locations (STN12).](image)

<table>
<thead>
<tr>
<th>Load Stage</th>
<th>(\sigma_s) (MPa)</th>
<th>Width of 1st crack (mm)</th>
<th>Width of 2nd crack (mm)</th>
<th>Width of 3rd crack (mm)</th>
<th>Width of 4th crack (mm)</th>
<th>Width of 5th crack (mm)</th>
<th>Average crack width (mm)</th>
<th>Maximum crack width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After 1st</td>
<td>158</td>
<td>0.075</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td>After 2nd</td>
<td>154</td>
<td>0.075</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.063</td>
<td>0.075</td>
</tr>
<tr>
<td>After 3rd</td>
<td>173</td>
<td>0.10</td>
<td>0.125</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>0.092</td>
<td>0.125</td>
</tr>
<tr>
<td>After 4th</td>
<td>212</td>
<td>0.10</td>
<td>0.25</td>
<td>0.10</td>
<td>0.05</td>
<td>-</td>
<td>0.125</td>
<td>0.25</td>
</tr>
<tr>
<td>After 5th</td>
<td>338</td>
<td>0.10</td>
<td>0.25</td>
<td>0.20</td>
<td>0.125</td>
<td>0.025</td>
<td>0.150</td>
<td>0.30</td>
</tr>
<tr>
<td>40kN</td>
<td>354</td>
<td>0.15</td>
<td>0.30</td>
<td>0.25</td>
<td>0.125</td>
<td>0.1</td>
<td>0.185</td>
<td>0.30</td>
</tr>
<tr>
<td>50kN</td>
<td>442</td>
<td>0.125</td>
<td>0.375</td>
<td>0.25</td>
<td>0.15</td>
<td>0.1</td>
<td>0.200</td>
<td>0.375</td>
</tr>
</tbody>
</table>

4.2 STS12:

Load versus Average Axial Strain:

Specimen STS12 was identical to specimen STN12 except that it was tested 3½ weeks later at age 57 days when the drying shrinkage had increased to \(\varepsilon_{sh0} = 2.49 \times 10^{-6}\). For STS12, at the time of testing at age 57 days, \(E_c = 21600\) MPa (i.e. \(n = 9.26\)) and \(f_c = 2.15\) MPa. The creep coefficient associated with the initial period of shrinkage was \(\varphi = 1.13\). The calculated average strain in the specimen prior to the commencement of loading (accounting for the restraint provided by the reinforcement) is calculated using the age-adjusted effective modulus method (Gilbert, 1988) as

\[
\frac{\varepsilon_{sh0}}{1 + \bar{n} \rho} = -2.09 \times 10^{-6}
\]

where \(\bar{n} = E_s / E_c = 17.6\) and \(E_s = E_c / (1 + \chi \varphi) = 11350\) MPa is the age-adjusted effective modulus for the concrete.

The average strain versus applied load measured using LVDTs throughout the tests is plotted in Figure 12, together with the bare bar response for the specimen.
Figure 11: Average strain ($\varepsilon_{s,avg}$) versus load $P$ for STS12

The early shrinkage resulted in a significant reduction in the load at first cracking (13.0 kN compared to 21.1 kN for STN12). The response after first cracking was similar to STN12, with the tension stiffening strain gradually reducing as the applied load was increased. As for STN12, a total of 5 primary cracks occurred in the specimen.

Table 3 provides values of average axial strain and tension stiffening strain at selected values of total applied load $P$, together with the average force carried by the concrete and the steel, $P_{\text{conc}}$ and $P_{\text{steel}}$, respectively, within the prism.

<table>
<thead>
<tr>
<th>Applied load, $P$ (kN)</th>
<th>0</th>
<th>10</th>
<th>13.0</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average strain, $\varepsilon_{s,avg}$ ($\times 10^6$)</td>
<td>-209</td>
<td>-171</td>
<td>-154</td>
<td>347</td>
<td>934</td>
<td>1452</td>
<td>2243</td>
</tr>
<tr>
<td>Tension stiffening strain $\varepsilon_{ts}$ ($\times 10^6$)</td>
<td>209</td>
<td>613</td>
<td>729</td>
<td>538</td>
<td>393</td>
<td>318</td>
<td>191</td>
</tr>
<tr>
<td>Avge force in concrete, $P_{\text{conc}}$ (kN)</td>
<td>+4.7</td>
<td>13.9</td>
<td>16.5</td>
<td>12.2</td>
<td>8.9</td>
<td>7.2</td>
<td>4.3</td>
</tr>
<tr>
<td>Avge force in steel bar, $P_{\text{steel}}$ (kN)</td>
<td>-4.7</td>
<td>-3.9</td>
<td>-3.5</td>
<td>7.8</td>
<td>21.1</td>
<td>32.8</td>
<td>50.7</td>
</tr>
</tbody>
</table>

Variation of Steel Strains:

The variations of steel and concrete forces in the middle 600 mm of the prism at various stages of loading are shown in Figure 12. The steel force was obtained directly from the measured steel strains and the concrete force was obtained by subtracting the force in the steel at each strain gauge location from the total force applied to the specimen.
(a) Just before first cracking, $P = 13.0$ kN.

(b) Just after first cracking, $P = 11.1$ kN.

(c) Just before 2nd crack, $P = 15.0$ kN.

(d) Just after 2nd crack, $P = 13.3$ kN.

(e) Just before 4th crack, $P = 18.1$ kN.

(f) Just after 4th crack, $P = 15.9$ kN.

(g) At $P = 40.0$ kN.

(h) At $P = 50.0$ kN.

Figure 12: Variation of forces in steel and concrete at different stages of loading (STS12).
Crack Pattern and Crack Width:

The crack locations and crack numbers for STN12 are shown in Figure 13 and the crack widths at various stages of loading are given in Table 4, together with the steel stress ($\sigma_s$) at the crack at each loading stage.

![Figure 13: Crack numbers and locations (STS12).](image)

**Table 4:** Crack widths at different loading stage (STS12).

<table>
<thead>
<tr>
<th>Load Stage</th>
<th>$\sigma_s$ (MPa)</th>
<th>Width of 1st crack (mm)</th>
<th>Width of 2nd crack (mm)</th>
<th>Width of 3rd crack (mm)</th>
<th>Width of 4th crack (mm)</th>
<th>Width of 5th crack (mm)</th>
<th>Average crack width (mm)</th>
<th>Maximum crack width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After 1st</td>
<td>98</td>
<td>0.025</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>After 2nd</td>
<td>118</td>
<td>0.05</td>
<td>0.075</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.063</td>
<td>0.075</td>
</tr>
<tr>
<td>After 3rd</td>
<td>132</td>
<td>0.125</td>
<td>0.20</td>
<td>0.125</td>
<td>-</td>
<td>-</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>After 4th</td>
<td>141</td>
<td>0.15</td>
<td>0.25</td>
<td>0.15</td>
<td>0.15</td>
<td>-</td>
<td>0.175</td>
<td>0.25</td>
</tr>
<tr>
<td>After 5th</td>
<td>265</td>
<td>0.175</td>
<td>0.35</td>
<td>0.225</td>
<td>0.175</td>
<td>0.05</td>
<td>0.195</td>
<td>0.35</td>
</tr>
<tr>
<td>40kN</td>
<td>354</td>
<td>0.125</td>
<td>0.40</td>
<td>0.50</td>
<td>0.375</td>
<td>0.30</td>
<td>0.34</td>
<td>0.50</td>
</tr>
<tr>
<td>50kN</td>
<td>442</td>
<td>0.25</td>
<td>0.425</td>
<td>0.525</td>
<td>0.425</td>
<td>0.30</td>
<td>0.385</td>
<td>0.525</td>
</tr>
</tbody>
</table>

4.3 STN16:

**Load versus Average Axial Strain:**

Specimen STN16 was subjected to monotonically increasing deformation soon after the end of moist curing. The elastic modulus of concrete at the time of testing at age 32 days was $E_c = 22400$ MPa (and with $E_s = 200000$ MPa, $n = 8.93$) and the tensile strength of concrete was $f_t = 2.04$ MPa. Prior to loading, the measured drying shrinkage strain in the unreinforced companion member was small at $\varepsilon_{sh0} = -28 \times 10^{-6}$ and the calculated strain in the specimen (accounting for the restraint provided by the reinforcement – see Figure 2) was $-23 \times 10^{-6}$.

The average strain versus applied load measured using LVDTs throughout the tests is plotted in Figure 14, together with the bare bar response for the specimen.

As expected, the post-cracking behaviour of STN16 was similar to that of STN12. First cracking occurred at Point B when $P = P_{cr} = 23.0$ kN) and 5 cracks occurred during the tests (indicated by the numbered peaks in the curve in Figure 14).
**Figure 14:** Average strain ($\varepsilon_{x,avg}$) versus load $P$ for STN16

Table 5 provides values of average axial strain and tension stiffening strain at selected values of total applied load $P$, together with the average force carried by the concrete and the steel, $P_{\text{concrete}}$ and $P_{\text{steel}}$, respectively, within the prism.

<table>
<thead>
<tr>
<th>Applied load, $P$ (kN)</th>
<th>0</th>
<th>10</th>
<th>23.0</th>
<th>30</th>
<th>45</th>
<th>65</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average strain, $\varepsilon_{x,avg}$ (x $10^{-6}$)</td>
<td>-23</td>
<td>8.2</td>
<td>57</td>
<td>386</td>
<td>866</td>
<td>1440</td>
<td>1990</td>
</tr>
<tr>
<td>Tension stiffening strain $\varepsilon_t$ (x $10^{-6}$)</td>
<td>23</td>
<td>241</td>
<td>523</td>
<td>360</td>
<td>253</td>
<td>177</td>
<td>124</td>
</tr>
<tr>
<td>Average force in concrete $P_{\text{concrete}}$ (kN)</td>
<td>+0.92</td>
<td>9.67</td>
<td>20.7</td>
<td>14.5</td>
<td>10.2</td>
<td>7.1</td>
<td>5.0</td>
</tr>
<tr>
<td>Average force in steel bar $P_{\text{steel}}$ (kN)</td>
<td>-0.92</td>
<td>0.33</td>
<td>2.3</td>
<td>15.5</td>
<td>34.8</td>
<td>57.9</td>
<td>80.0</td>
</tr>
</tbody>
</table>

**Variation of Steel Strains:**

Figure 15 show the variation in the tensile forces carried by steel and concrete at different loading stages for STN16.

**Crack Pattern and Crack Width:**

The crack locations and crack numbers for STN12 are shown in Figure 16 and the crack widths at various stages of loading are given in Table 6, together with the steel stress ($\sigma_s$) at the crack at each loading stage.
(a) Just before first cracking, $P = 23.0$ kN.

(b) Just after first cracking, $P = 20.8$ kN.

(c) Just before 2nd crack, $P = 24.3$ kN.

(d) Just after 2nd crack, $P = 22.7$ kN.

(e) Just before 3rd crack, $P = 24.9$ kN.

(f) Just after 3rd crack, $P = 22.8$ kN.

(g) At $P = 40.0$ kN.

(h) At $P = 75.0$ kN.

Figure 15: Variation of forces in steel and concrete at different stages of loading (STN16).
Table 6: Crack widths at different loading stage (STN16).

<table>
<thead>
<tr>
<th>Load Stage</th>
<th>$\sigma_c$ (MPa)</th>
<th>Width of 1st crack (mm)</th>
<th>Width of 2nd crack (mm)</th>
<th>Width of 3rd crack (mm)</th>
<th>Width of 4th crack (mm)</th>
<th>Width of 5th crack (mm)</th>
<th>Average crack width (mm)</th>
<th>Maximum crack width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After 1st</td>
<td>104</td>
<td>0.013</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>After 2nd</td>
<td>113</td>
<td>0.025</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0375</td>
<td>0.05</td>
</tr>
<tr>
<td>After 3rd</td>
<td>114</td>
<td>0.05</td>
<td>0.075</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>0.0583</td>
<td>0.075</td>
</tr>
<tr>
<td>After 4th</td>
<td>196</td>
<td>0.10</td>
<td>0.25</td>
<td>0.125</td>
<td>0.05</td>
<td>-</td>
<td>0.1313</td>
<td>0.25</td>
</tr>
<tr>
<td>After 5th</td>
<td>299</td>
<td>0.125</td>
<td>0.325</td>
<td>0.10</td>
<td>0.225</td>
<td>0.025</td>
<td>0.160</td>
<td>0.325</td>
</tr>
<tr>
<td>75 kN</td>
<td>373</td>
<td>0.15</td>
<td>0.325</td>
<td>0.15</td>
<td>0.125</td>
<td>0.125</td>
<td>0.215</td>
<td>0.325</td>
</tr>
<tr>
<td>90 kN</td>
<td>447</td>
<td>0.20</td>
<td>0.375</td>
<td>0.25</td>
<td>0.15</td>
<td>0.15</td>
<td>0.245</td>
<td>0.375</td>
</tr>
<tr>
<td>105 kN</td>
<td>522</td>
<td>0.25</td>
<td>0.50</td>
<td>0.25</td>
<td>0.275</td>
<td>0.225</td>
<td>0.3</td>
<td>0.50</td>
</tr>
</tbody>
</table>

4.4 STS16:

Load versus Average Axial Strain:

Specimen STS12 was identical to specimen STN12 except that it was tested 3½ weeks later at age 57 days when the drying shrinkage had increased to $\varepsilon_{sh0} = -249 \times 10^{-6}$. For STS16, at the time of testing at age 57 days, $E_c = 21600$ MPa (i.e. $n = 9.26$) and $f_r = 2.15$ MPa. The creep coefficient associated with the initial period of shrinkage was $\phi = 1.13$. The calculated average strain in the specimen prior to the commencement of loading (accounting for the restraint provided by the reinforcement) is calculated using the age-adjusted effective modulus method (Gilbert, 1988) as

$$\frac{\varepsilon_{sh0}}{1 + \bar{n}^2 \rho} = -185 \times 10^{-6}$$

where $\bar{n}^2 = E_e / E_c = 17.6$ and $E_c = E_c / (1 + n \phi) = 11350$ MPa is the age-adjusted effective modulus for the concrete.

The average strain versus applied load measured using LVDTs throughout the tests is plotted in Figure 17, together with the bare bar response for the specimen.
Figure 17: Average strain ($\varepsilon_{avg}$) versus load $P$ for STS12

The early shrinkage resulted in a significant reduction in the load at first cracking (11.6 kN compared to 23.0 kN for STN16). The response after first cracking was similar to STN16, with the tension stiffening strain gradually reducing as the applied load was increased. A total of 6 primary cracks occurred in the specimen.

Table 3 provides values of average axial strain and tension stiffening strain at selected values of total applied load $P$, together with the average force carried by the concrete and the steel, $P_{conc}$ and $P_{steel}$, respectively, within the prism.

<table>
<thead>
<tr>
<th>Applied load, $P$ (kN)</th>
<th>0</th>
<th>10</th>
<th>11.6</th>
<th>20</th>
<th>30</th>
<th>45</th>
<th>65</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average strain, $\varepsilon_{avg}$ (x 10^-6)</td>
<td>-185</td>
<td>-149</td>
<td>-140</td>
<td>179</td>
<td>469</td>
<td>879</td>
<td>1461</td>
<td>2049</td>
</tr>
<tr>
<td>Tension stiffening strain $\varepsilon_t$ (x 10^-6)</td>
<td>185</td>
<td>398</td>
<td>429</td>
<td>319</td>
<td>277</td>
<td>240</td>
<td>156</td>
<td>65</td>
</tr>
<tr>
<td>Avge force in concrete, $P_{conc}$ (kN)</td>
<td>7.4</td>
<td>16.0</td>
<td>17.2</td>
<td>12.8</td>
<td>11.1</td>
<td>9.7</td>
<td>6.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Avge force in steel bar, $P_{steel}$ (kN)</td>
<td>-7.4</td>
<td>-6.0</td>
<td>-5.6</td>
<td>7.2</td>
<td>18.9</td>
<td>35.3</td>
<td>58.7</td>
<td>82.4</td>
</tr>
</tbody>
</table>

Variation of Steel Strains:

The variations of steel and concrete forces in the middle 600 mm of the prism at various stages of loading are shown in Figure 18. The steel force was obtained directly from the measured steel strains and the concrete force was obtained by subtracting the force in the steel at each strain gauge location from the total force applied to the specimen.
Figure 18: Variation of forces in steel and concrete at different stages of loading (STS16).
Crack Pattern and Crack Width:

The crack locations and crack numbers for STS16 are shown in Figure 19 and the crack widths at various stages of loading are given in Table 8, together with the steel stress (\(\sigma_s\)) at the crack at each loading stage.

![Figure 19: Crack numbers and locations (STS16).](image)

<table>
<thead>
<tr>
<th>Load Stage</th>
<th>(\sigma_s) (MPa)</th>
<th>Crack width (mm)</th>
<th>Average crack width (mm)</th>
<th>Maximum crack width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After 1st</td>
<td>55</td>
<td>0.05 - - - - -</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>After 2nd</td>
<td>63</td>
<td>0.05 0.025 - - -</td>
<td>0.038</td>
<td>0.05</td>
</tr>
<tr>
<td>After 3rd</td>
<td>65</td>
<td>0.075 0.05 0.05 -</td>
<td>0.058</td>
<td>0.075</td>
</tr>
<tr>
<td>After 4th</td>
<td>78</td>
<td>0.075 0.05 0.05 0.05 -</td>
<td>0.056</td>
<td>0.075</td>
</tr>
<tr>
<td>After 5th</td>
<td>82</td>
<td>0.125 0.10 0.125 0.125 0.15 -</td>
<td>0.125</td>
<td>0.15</td>
</tr>
<tr>
<td>After 6th</td>
<td>136</td>
<td>0.125 0.125 0.125 0.15 0.175 0.025</td>
<td>0.121</td>
<td>0.175</td>
</tr>
<tr>
<td>50 kN</td>
<td>249</td>
<td>0.15 0.125 0.15 0.20 0.325 0.05</td>
<td>0.167</td>
<td>0.325</td>
</tr>
<tr>
<td>75 kN</td>
<td>373</td>
<td>0.175 0.10 0.25 0.325 0.375 0.25</td>
<td>0.246</td>
<td>0.375</td>
</tr>
<tr>
<td>90 kN</td>
<td>448</td>
<td>0.175 0.125 0.325 0.375 0.425 0.30</td>
<td>0.288</td>
<td>0.425</td>
</tr>
</tbody>
</table>

4.5 Comparisons and Further Discussion

The curves in Figure 20 show the average concrete tensile stress versus the average axial strain for each test specimen. The dot at the left-hand end of each curve in Figures 20a and 20b signifies the situation at the beginning of loading, where an initial tensile stress existed in the concrete due to shrinkage (\(\varepsilon_{\text{sh}}\)) prior to the application of the external load \(P\). The tensile strength of concrete is a highly variable material property and, in these four specimens, the concrete tensile stress at first cracking was 2.00 MPa and 2.11 MPa for STN12 and STN16, respectively, and 1.67 MPa and 1.76 MPa for STS12 and STS16, respectively. The concrete in both specimens with significant initial shrinkage (STS12 and STS16) had a significantly lower tensile strength than the concrete in the other two specimens (despite being almost 28 days older at the time of testing) and the average tensile stress after cracking under normal in-service conditions (\(\varepsilon_{\text{avg}} < 1600 \times 10^{-6}\)) was also lower.
Figure 20: Average concrete tensile stress versus axial strain.

After initial cracking, the average concrete stress in all specimens gradually reduced as the deformation increased. It would appear that the concrete tensile stress (in effect the tension stiffening) decreases at a slower rate if significant shrinkage occurs prior to first cracking, but further testing is required to gain confidence in this conclusion.

The average concrete tensile stress versus axial strain curves for STN12 and STN16 (Figure 20c) are remarkably similar, as are the curves for STS12 and STS16 (Figure 20d). While the tensile stress in the concrete after cracking appears to be not significantly affected by the bar diameter (and reinforcement ratio $\rho = A_s/A_c$), the contribution of the cracked tensile concrete to the stiffness of the member is a larger proportion of the total stiffness when the reinforcement ratio is smaller.

This is illustrated in Figure 21 where the tension stiffening strain is plotted against the average axial strain caused by load. After cracking, the tension stiffening strain in STN12 ($\varepsilon_{ts,12}$) is significantly greater than in STN16 ($\varepsilon_{ts,16}$), with the ratio ($\varepsilon_{ts,12}/\varepsilon_{ts,16}$) under service conditions ($\varepsilon_{ts,avg} < 1600 \times 10^{-6}$) in the range 1.7 to 1.8. It is noted that the inverse of the ratio of the reinforcement ratios for each specimen (i.e. $\rho_{t1}/\rho_{t2}$) is 1.79. It appears that the tension stiffening strain is almost inversely proportional to the reinforcement ratio, with tension stiffening becoming more significant as the reinforcement ratio decreases.
Figure 21: Tension stiffening strain versus axial strain (STN12 and STN16).

The maximum crack widths given in Tables 2, 4, 6 and 8 are plotted against steel stress at each crack in Figure 22 and a line of best fit for each specimen is shown. It appears that shrinkage strain in the concrete prior to cracking has a significant effect on maximum crack widths (as shown in Figure 22a and 22b). For a particular steel stress, the crack widths are

Figure 22: Steel stress versus maximum crack width.
significantly greater when the specimen has been exposed to significant shrinkage prior to loading. The effect of changing the reinforcement ratio on crack widths is shown in Figures 22c and 22d. Figure 22c compares the steel stress versus maximum crack width curves for the two specimens without significant shrinkage. From these tests it appears that reinforcement ratio has little effect. However, for the two specimens with significant initial shrinkage the specimen with the smaller reinforcement ratio has significantly wider cracks. It is felt that additional testing is required to gain confidence in these conclusions.

5. LONG-TERM TEST RESULTS AND DISCUSSION

For the long-term test, the average strain was measured using Demec gauges attached on the opposite sides of the concrete surface at pre-selected times throughout the duration of the tests. On each side, 5 Demec points were fixed to the concrete surface at 250mm centres and 4 readings were taken at each time instant. The readings were averaged to obtain the average axial strain of the specimen (i.e. the sum of the elastic, creep and shrinkage strains in the concrete).

Specimen LTN12A was initially loaded up to 40kN, at which point 5 cracks had developed, and this load level was then held constant for the duration of the test. Specimen LTN12B was loaded up to 20kN, at which point only one crack had developed, and this load level was then held constant. During the period of sustained loading, 3 additional cracks developed in LTN12B due primarily due to the restraint to drying shrinkage provided by the reinforcing bar. In specimen LTN12B, at the formation of each additional crack, the tension stiffening effect reduced appreciably.

Both specimens LTN12A and LTN12B were moist cured until two days before the test started (at age 33 days). The specimens were initially loaded in 5 kN increments up to a load $P = 40$ kN. The load was then sustained for a period of 50 days (i.e. from age 33 days to age 83 days). The measured elastic modulus of concrete at the time of first loading was $E_e = 22400$ MPa ($n = 8.93$) and the tensile strength of concrete was $f_t = 2.04$ MPa. The measured shrinkage strain in the concrete just before first loading was $-52 \times 10^{-6}$ and the creep coefficient for this initial period of drying (up to age 33 days) was 0.3 (i.e. $E_e = 18060$ MPa, $n' = 11.1$). The initial concrete tensile stress caused by restraint to shrinkage prior to loading is $\sigma_{st0} = 0.11$ MPa.

The shrinkage strain at the end of the sustained load period was $-310 \times 10^{-6}$. The creep coefficient associated with the sustained load period was 1.38.

5.1 Load versus Average Axial Strain:

Figures 23 and 24 trace the load versus average strain response for LTN12A and LTN12B, respectively. Note that before each long-term test commenced, the initial drying shrinkage strain was $\varepsilon_{sh0} = -52 \times 10^{-6}$ and, accounting for restraint provided by the reinforcing bar, the strain in the concrete was $-46 \times 10^{-6}$.
For specimen LTN12A, the average axial strain immediately after application of the 40 kN load was $\varepsilon_{\text{ave}}(0) = 1409 \times 10^{-6}$ and the tension stiffening strain was $\varepsilon_t(0) = 361 \times 10^{-6}$. The average stress in the steel is therefore $\sigma_s(0) = E_s \varepsilon_{\text{ave}}(0) = 282$ MPa and the average force carried by the steel bar is $T_s(0) = A_s \sigma_s(0) = 31.84$ kN. This means the average tensile force in the cracked concrete is $T_c(0) = P - T_s(0) = 8.16$ kN and the average concrete stress is $\sigma_c(0) = T_c(0)/A_c = 0.825$ MPa. After 50 days under load, the average axial strain had increased to $\varepsilon_{\text{ave}}(50) = 1556 \times 10^{-6}$ and the tension stiffening strain had reduced to $\varepsilon_t(50) = 214 \times 10^{-6}$. The corresponding steel and concrete forces and stresses are $\sigma_s(50) = E_s \varepsilon_{\text{ave}}(50) = 311$ MPa; $T_s(50) = A_s \sigma_s(50) = 35.17$ kN; $T_c(50) = P - T_s(50) = 4.83$ kN and the average concrete stress is $\sigma_c(50) = T_c(50)/A_c = 0.49$ MPa. It appears that for specimen LTN12A, initially with a fully developed crack pattern, the average tensile stress in the concrete and the tension stiffening strain reduced to 59.4% of their initial value in the first 50 days under sustained load.
For specimen LTN12B, the average axial strain immediately after application of the 20 kN load was $\varepsilon_{\text{ave}, y}(0) = 89 \times 10^{-6}$ and the tension stiffening strain was $\varepsilon_t(0) = 796 \times 10^{-6}$. The average stress in the steel is therefore $\sigma_s(0) = E_s E_s, \varepsilon_{\text{ave}, y}(0) = 17.8 \text{ MPa}$ and the average force carried by the steel bar is $T_s(0) = A_s \sigma_y(0) = 2.01 \text{ kN}$. This means the average tensile force in the cracked concrete is $T_c(0) = P - T_s(0) = 17.99 \text{ kN}$ and the average concrete stress is $\sigma_c(0) = T_c(0)/A_c = 1.82 \text{ MPa}$. At this time, only one crack had developed. After 50 days under load, the average axial strain had increased to $\varepsilon_{\text{ave}, y}(50) = 239 \times 10^{-6}$ and the tension stiffening strain had reduced to $\varepsilon_t(50) = 646 \times 10^{-6}$. The corresponding steel and concrete forces and stresses are $\sigma_s(50) = E_s, \varepsilon_{\text{ave}, y}(50) = 47.8 \text{ MPa}$; $T_s(50) = A_s \sigma_y(50) = 5.40 \text{ kN}$; $T_c(50) = P - T_s(50) = 14.59 \text{ kN}$ and the average concrete stress is $\sigma_c(50) = T_c(50)/A_c = 1.48 \text{ MPa}$. Several additional cracks occurred with time due to restraint to shrinkage. For this relatively lightly loaded member, the average tensile stress in the concrete and the tension stiffening strain reduced to 81.2% of their initial value in the first 50 days under sustained load.

### 5.2 Variation of Stresses:

The variation of steel strains over a 600mm gauge length of the reinforcing bar were measured using strain gauges as the time under load increased and the variation in force in the reinforcing steel was determined from the measured strains. The variation in concrete force was obtained by subtracting the force in the steel at each strain gauge location from the total force applied to the specimen.

Figures 25 and 26 show the variation in the tensile stresses carried by concrete and steel immediately after the application of the full sustained load and after 50 days under the sustained load for specimens LTN12A and LTN12B, respectively.

![Figure 25: Initial and final concrete and steel stresses LTN12A (P = 40 kN).](image_url)
Figure 26: Initial and final concrete and steel stresses LTN12B \((P = 20 \text{ kN})\).

5.3 Crack Locations and Crack Widths:

The crack pattern and crack width of both long-term tests are shown in Figure 27, both immediately after loading and after 50 days under sustained load.

Figure 27: Crack numbers and crack locations (LTN12A and LTN12B).
For specimen LTN12A, crack number 4 was the widest crack, with a maximum crack width of 0.375 mm immediately after loading ($P = 40$ kN) and 0.675 mm after 50 days under load. The average crack width was 0.24 mm immediately after loading and 0.335 mm after 50 days under load. For specimen LTN12B, the width of the only crack immediately after first loading ($P = 20$ kN) was 0.075 mm and this increased to 0.125 mm after 50 days under load. The average crack width after 50 days (4 cracks total) was 0.081 mm.

6. CONCLUDING REMARKS

An experimental study of the time-dependent stiffness of cracked tension members has been presented. The mechanisms of tension stiffening have been explained and the results of short-term and long-term tests have been presented. It has been demonstrated that shrinkage, both before and after loading, has a profound affect on the load-deformation response of reinforced concrete in tension. Shrinkage before first loading is restrained by the embedded reinforcement and a tensile restraining force is imposed on the concrete before any external load is applied. This restraining force reduces the external load required to cause cracking. Shrinkage causes wider cracks and a reduction in tension stiffening with time. Consideration of drying shrinkage is the key to understanding the deformational response of reinforced concrete in tension under in-service conditions.

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8. REFERENCES

