BUCKLING OF REINFORCED CONCRETE WALLS

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Buckling, columns; core walls, high strength concrete; reinforced concrete; shear walls, stability, structural analysis; structural design.

An approximate buckling analysis for simply supported reinforced concrete walls under axial compression is derived. It is assumed that the eccentricity of the axial loading is always within the middle third of the wall thickness. Based on a stress-strain relationship for normal to high strength concretes, a formula for the critical wall slenderness is presented. The buckling load is then used to estimate the amplified moments in the longitudinal and transverse directions. An upper limit on the amplification factor is determined. Formulae for the minimum percentage reinforcement and yield moment capacity in the transverse direction are derived. It is found that the critical direction for reinforcement is not necessarily in the direction of loading since the moment capacity is increased due to the presence of the axial load. An example of a core wall design is also presented.
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by Mario M. Attard

1. INTRODUCTION

The majority of multistorey structures built in Australia have reinforced concrete cores. These cores are designed to carry either the whole or part of the lateral loading (usually wind shear), as well as the dead and live loads. The cores normally form the main structural element through which the lifts move. Each building has its own unique core layout. Figure 1 shows one example of a core layout. The central diaphragm walls in this layout, span horizontally approximately 8 m. These diaphragm walls are only restrained by the intersecting walls along their sides (there are no floors intersecting these walls). The critical slenderness of these walls which run the total length of the building, is therefore the horizontal span to thickness ratio. As with columns in the lower storeys, economic benefits can be gained by reducing the core wall thickness through the use of high strength concrete. Reducing the wall thickness however, results in walls of greater slenderness which require an assessment of the amplification effects of buckling. The present design methods in the Australian Standard for Concrete Structures AS3600, are very conservative and are applicable to walls which buckle in predominantly one-way bending. The rules place a limit on the slenderness ratio of walls of 30. To take full advantage of the use of high strength concrete, a more detailed assessment of the effects of buckling are required incorporating the two-way nature of bending in core walls.

2. DESIGN OF WALLS UNDER ONE-WAY ACTION

For walls in one-way bending (top and bottom edges simply supported and vertical edges free), AS3600 gives the design axial strength per unit length $N_u$ of a braced wall in compression as

$$\phi N_u = \Phi 0.6 f'_c (t_w - 1.2e - 2e_n)$$

(1)

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where $f_c$ is the compressive strength of concrete, $\phi$ can be taken as 0.6, $t_w$ is the wall thickness, $e$ is the load eccentricity which can be taken as 0.05 $t_w$ and $e_a$ an additional eccentricity defined as

$$e_a = \frac{(H_{we})^2}{2500t_w}$$ (2)

with $H_{we}$ the effective height of the wall (for walls restrained against rotation at both ends $H_{we} = 0.75$ times the unsupported height of the wall, $H$). Equation 1 can be rewritten in the form

$$\phi N_u = \phi 0.6 f'_c (t_w - 1.2e - 2e_a)$$

$$= \phi 0.6 f'_c \left( t_w - 1.2 \times 0.05 t_w - \frac{2H_{we}^2}{2500t_w} \right)$$

$$= \phi 0.564 f'_c t_w \left( 1 - \left( \frac{H_{we}}{34.3t_w} \right)^2 \right)$$ (3)

This is very similar to the ACI 318-1989 Code formula given by

$$\phi N_u = \phi 0.55 f'_c t_w \left( 1 - \left( \frac{kH}{32t_w} \right)^2 \right)$$ (4)

with $\phi$ taken as 0.7 and $k$ equal to 0.8 for walls braced top and bottom against lateral translation and restrained against rotation at one or both ends. Equation 4 is empirically based on the work of several researchers including Kripinarayanan (1977), Pillai and Parthasarathy (1977) and, Saheb and Desai (1990). Equation 3 is plotted in Fig. 2.

In the limit as the height to thickness ratio approaches zero, i.e. no slenderness effects, the axial load capacity is defined by $\phi 0.55 f'_c A_g$ where $A_g$ is the cross-sectional area of the wall. For walls, the axial load is assumed to be applied within the middle third of the thickness (no tension). The maximum eccentricity is therefore $\frac{t_w}{6} = 0.167t_w$. At this eccentricity, the axial
load capacity is approximately $0.6f'_c A_g$ (see Fig. 3), agreeing with Eqn 3 when the slenderness is zero. This eccentricity is much larger (3.34 times) than the minimum eccentricity $0.05t_w$, considered for columns.

A conservative design procedure for core walls as outlined by Mendis et al (1993), is to calculate a fictitious ultimate stress capacity by dividing Eqn 3 by the wall thickness and comparing this quantity to a stress calculated using factored ultimate loads and formulae for stresses based on elastic beam theory. This procedure is extremely conservative. Figure 4 compares the AS3600 wall axial load capacity formula (Eqn 3) against available experimental results for simply supported walls. It is seen that the AS3600 formula is extremely conservative for walls simply supported on all four sides.

3. BUCKLING OF SIMPLY SUPPORTED REINFORCED CONCRETE WALLS

The walls making up a typical core are subjected to predominantly axial force loading, as well as some in-plane bending and shear. The shear cores can be highly redundant and hence the best approach for the analysis of stresses is to use computer based numerical analysis such as the finite element technique. The central diaphragm wall of the core layout shown in Fig. 1, is restrained along the side edges by intersecting walls which are themselves braced by floors at each level of the building. If the diaphragm is very thin, ie approaching the slenderness limit in AS3600 of 30 the slenderness effects will magnify any moment due to the eccentricity of the loads and geometric imperfections present in the walls. A buckling analysis is important as it can be used to estimate the amplification of these moments due to slenderness effects. A finite element package for the buckling analysis of reinforced concrete walls is presently being developed at the University of New South Wales. The first stage of this package is complete and is currently being verified against available experimental data.

An approximate analysis for walls simply supported on all four sides loaded by uniform axial forces per unit length $N_x$ along two opposite sides, will be outlined here (see Fig. 5). It is assumed here that the wall is always in compression. The procedures developed can be applied to other boundary conditions. The aim is to estimate the buckling load of a reinforced concrete wall and determine the amplified moments. The design axial force and amplified moments can
then be checked against the ultimate capacity of the wall section through the use of an interaction diagram.

Consider a wall with sides b and a, as shown in Fig. 5. A rectangular axis system is defined with the origin at the left top corner. The wall is initially considered to be perfectly straight and in a state of equilibrium. To ascertain whether the wall is stable small out-of-plane bending perturbations are applied. The associated equation of out-of-plane bending equilibrium assuming an orthotropic material is

\[
D_{xx} \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{yy} \frac{\partial^4 w}{\partial y^4} = -N_x \frac{\partial^2 w}{\partial x^2}
\]  

(5)

in which \(w\) is the out-of-plane bending perturbation and \(D_{xx}\) and \(D_{yy}\) are the bending tangent rigidities in the \(x\) and \(y\) directions, respectively, defined by

\[
D_{xx} = \frac{E_{xx}}{(1-v^2)} \frac{l_w^3}{12}
\]

(6)

\[
D_{yy} = \frac{E_{yy}}{(1-v^2)} \frac{l_w^3}{12}
\]

(7)

In the above \(E_{xx}\) and \(E_{yy}\) are the tangent modulus of the material in the \(x\) and \(y\) directions, respectively. The term \(v\) is the Poisson's ratio which for concrete is approximately 0.2 in the proportional range. The variation of the dilation (Poisson's ratio) with stress is ignored here. \(D_{xyt}\) is the warping tangent rigidity and can be taken as

\[
D_{xyt} = \frac{E_{xx} E_{yy} l_w^3}{(1-v^2)} \frac{l_w^3}{12}
\]

(8)

Assuming the wall load is approximately uniform and concentric, for simply supported edges the axial stresses will be uniform throughout the entire wall before buckling. To simplify the present derivation, the reinforcement in the wall will be ignored. For concrete the stress-strain relationship is nonlinear (refer to Fig. 6). As the load in the wall is increased from zero, the
tangent modulus in the axial direction \(x\) changes. The tangent modulus in the \(y\) direction depends on the level of bending in the \(y\) direction (about the \(x\) axis) and will vary across the width of the wall. Since there is no axial force in this direction it is likely that any bending will cause cracking. If this moment is less than the yield moment than the tangent modulus and the secant modulus in this direction can be set to approximately \(0.4 E_c\). Hence we can write

\[
E_{st} = E_t \quad \text{and} \quad E_{sy} = 0.4 E_c \tag{9}
\]

The tangent modulus in the \(x\) direction can be obtained from the stress-strain relationship for concrete.

Attard (1994) suggested the following equation for the stress-strain curve of concretes applicable to strengths between 20 to 120 MPa.

\[
Y = \frac{AX + BX^2}{1 + (A - 2)X + (B + 1)X^2} \quad \text{where} \quad Y = \frac{f}{f_c}, \quad X = \frac{\varepsilon}{\varepsilon_c} \tag{10}
\]

\[
\text{and} \quad \forall X \geq 0 \quad 0 \leq Y \leq 1
\]

In the above equation, \(f\) is the stress at strain \(\varepsilon\) while \(f_c\) is the peak uniaxial stress at strain \(\varepsilon_c\). The strain \(\varepsilon_c\) is given by

\[
\varepsilon_c = \frac{f_c}{E_c} \frac{4.26}{\sqrt[4]{f_c}} \quad \text{Crushed Aggregates} \tag{11}
\]

while for gravel aggregates the following was proposed :

\[
\varepsilon_c = \frac{f_c}{E_c} \frac{3.78}{\sqrt[4]{f_c}} \quad \text{Gravel Aggregates} \tag{12}
\]

Two sets of the constants \(A\) and \(B\) are required. One for the ascending and another for the descending portions of the curve. For the ascending curve the constants are given by
\[
A = \frac{E_0}{f_c} \varepsilon_c \\
B = \frac{(A - 1)^2}{1 - \frac{f_{pl}}{f_c}} + \frac{A^2(1 - \alpha)}{\alpha^2 \frac{f_{pl}}{f_c} (1 - \frac{f_{pl}}{f_c})} - 1
\]

where \(\alpha = \frac{E_0}{E_c}\), \(E_0\) is the initial tangent modulus and \(E_c\) is the secant modulus measured at a stress of \(f_{pl}\) (usually 0.45 \(f_c\)). The initial tangent modulus can be assumed to be vary linearly between \(1.17E_c\) and \(E_c\) for 20 MPa and 100 MPa concretes, respectively. The constants for the descending curve are

\[
A = \frac{f_{ic}}{f_c} \frac{(\varepsilon_{ic} - \varepsilon_c)^2}{\varepsilon_c \varepsilon_{ic} f_c - f_{ic}} \\
B = 0
\]

where the symbols \(\varepsilon_{ic}, f_{ic}\) denote the coordinates of the inflection point on the descending curve approximated by

\[
\frac{\varepsilon_{ic}}{\varepsilon_c} = 2.5 - 0.3 \ln(f_c) \\
\frac{f_{ic}}{f_c} = 1.41 - 0.17 \ln(f_c)
\]

Consequently, in order to establish the full uniaxial stress-strain relationship the parameters required are the uniaxial compressive strength and the secant elastic modulus. The formula for the tangent modulus obtained by differentiating Eqn 10 and is

\[
E_t = -\frac{X^2(A + 2B) - 2BX - A}{X^2(B + 1) + X(A - 2) + 1} \frac{f_s}{\varepsilon_c}
\]

The axial equilibrium in a wall under an axial force \(N_x\) is given by the following

\[
N_x = E_{sec} \varepsilon t_w
\]

where \(E_{sec}\) is the secant modulus at a strain \(\varepsilon\). Figures 7 and 8 show the variation of the secant and tangent modulus with stress level for different grades of concrete. The secant modulus is fairly constant and equal to the elastic modulus up to the proportional limit. This is as expected. After the proportional limit, the secant modulus reduces gradually to about 0.6\(E_c\) at
the peak stress. The rate of change of the tangent modulus is much greater than the secant modulus and reduces to zero at the peak stress. The tangent modulus for high strength concretes is greater than for normal strength concretes. Equation 5 can be used to determine the buckling load. The fundamental buckled shape for a simply supported plate is

\[ w(x, y) = C_1 \sin \left( \frac{mx}{a} \right) \sin \left( \frac{ny}{b} \right) \]  

(18)

where \( C_1 \) is an undetermined constant and \( m \) is an integer representing the number of half waves into which the plate buckles. The solution to the buckling load of an orthotropic plate can then be written as

\[ N_{cr} = \frac{k\pi^2 E_{xt} t_w^2}{12(1 - v^2)} \frac{l_w^4}{b^2} \]  

(19)

where \( k \) is a buckling coefficient and is given by

\[ k = \left\{ \frac{m^2}{(a/b)^2} + 2 \sqrt{\frac{E_{yt}}{E_{xt}}} \frac{E_{yt}}{E_{xt}} \left( \frac{a/b}{m} \right)^2 \right\} \]  

(20)

When \( E_{yt} \) is zero the solution of Eqn 19 is that of a simply supported column of length "a". The minimum \( k \) can be found by differentiating Eqn 20 with respect to \( m \) and setting the resulting equation to zero. The minimum \( k \) is therefore

\[ a t m = \frac{E_{yt}}{E_{xt}} \frac{a}{b} \quad k_{min} = 4 \sqrt{\frac{E_{yt}}{E_{xt}}} \]  

(21)

Equating Eqn 19 to 17 we have for the critical slenderness

\[ \left( \frac{b}{t_w} \right)^2 = \frac{k\pi^2}{12(1 - v^2)} \frac{E_{xt}}{E_{sec}} \frac{1}{\varepsilon'} \]  

(22)
For any value of strain, Eqn 22 can be used to calculate the critical slenderness as well as the corresponding buckling stress from the stress-strain relationship. Setting the upper limit on the axial capacity to 0.85 $\bar{f}_c$ $t_w$, Fig. 9 plots the relationship between the buckling load of a simply supported wall and the wall slenderness. Also plotted in Fig. 9 are available experimental results for the capacity of simply supported walls. The experimental results of Saheb and Desay (1990) are for end loads applied at an eccentricity of $t_w/6$ and hence have a much lower capacity than the buckling load.

4. AMPLIFIED MOMENTS

The buckling load can be used to estimate the amplified moments. The bending moments per unit length about the y and x axes are denoted by $M_{xx}$ and $M_{yy}$, respectively. From plate theory, the bending moments are related to the plate curvatures by

$$M_{xx} = \frac{E_h t^3}{12(1-\nu^2)} \left[ \frac{\partial^2 w}{\partial x^2} + \nu \sqrt{\frac{0.4E_h}{E_{sec}}} \frac{\partial^2 w}{\partial y^2} \right]$$

$$M_{yy} = \frac{0.4E_h t^3}{12(1-\nu^2)} \left[ \frac{\partial^2 w}{\partial y^2} + \nu \sqrt{\frac{0.4E_h}{E_{sec}}} \frac{\partial^2 w}{\partial x^2} \right]$$

(23)

The moment in the y direction is assumed to be less than the yield moment. If initial imperfections exist in the wall than the axial end loads will produce bending from the commencement of loading. Let's assume the wall has a imperfection profile $w_i(x,y)$ similar to the critical buckling mode and let's set the maximum out-of-plane imperfection to $\bar{w}_o$. From the solution to the buckled shape of a plate we set

$$w_o(x,y) = \bar{w}_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

(24)

For a design axial force per unit length of $N_x^*$ the amplified bending deflection $w(x,y)$ can be obtained from
\[ w(x, y) = w_0(x, y) \delta_b \]  
\[(25)\]

where \( \delta_b \) is the amplification or magnifier factor defined by

\[
\delta_b = \frac{1}{1 - \frac{N_0^*}{N_{cr}(1 + \beta_d)}}
\]
\[(26)\]

Note in the design of columns in AS3600 a \( \phi \) factor of 0.6 is applied to the buckling load. This has not been incorporated in Eqn 26. The buckling load given in Fig. 9 has already been reduced by a factor of 0.85 and is further reduced in the above equation by the incorporation of the creep factor \( \beta_d \). The creep factor \( \beta_d \) is the ratio of the dead load to the total load and takes account of the reduction of the buckling load due to creep under sustained load. Substituting Eqns 24, 25 and 26 into Eqn 23 gives the amplified design moments. The maximum moments occur when \( x = \frac{a}{2m} \) and \( y = \frac{b}{2} \) and are given by:

\[
M_{xx}^* = \delta_b \frac{E_{sec} t_w^3}{12(1 - \nu^2)} \left\{ \left( \frac{ma}{b} \right)^2 + \nu \left( \frac{0.4E_c}{E_{sec}} \left( \frac{\pi}{b} \right)^2 \right) \right\} \bar{w}_o
\]
\[(27)\]

\[
M_{yy}^* = \delta_b \frac{0.4E_c t_w^3}{12(1 - \nu^2)} \left\{ \left( \frac{\pi}{b} \right)^2 + \nu \left( \frac{E_{sec}}{0.4E_c} \left( \frac{ma}{b} \right)^2 \right) \right\} \bar{w}_o
\]

Using Eqn 21 the above equations reduce to

\[
M_{xx}^* = \delta_b \frac{E_{sec} t_w^3}{12(1 - \nu^2)} \left( \frac{\pi}{b} \right)^2 \sqrt{\frac{0.4E_c}{E_{sec}}} \left\{ 1 + \nu \right\} \bar{w}_o
\]
\[(28)\]

\[
M_{yy}^* = \delta_b \frac{0.4E_c t_w^3}{12(1 - \nu^2)} \left( \frac{\pi}{b} \right)^2 \left\{ 1 + \nu \right\} \bar{w}_o
\]

An estimate of the initial lateral imperfection \( \bar{w}_o \) of the wall can be made from the formula for the deflection of a one-way plate under end moments \( M_0 \), that is

\[
\bar{w}_o = \frac{M_0 b^2}{8D}
\]
\[(29)\]
where \( D \) is the bending rigidity. Substituting this equation into Eqn 30 gives

\[
M_{xx}^* = \delta_b M_o \frac{\pi^2}{8} \sqrt{\frac{0.4E_{sec}}{E_c}} \left\{ 1 + \nu \right\}
\]

\[
M_{yy}^* = \delta_b M_o 0.05\pi^2 \left\{ 1 + \nu \right\}
\]

(30)

The initial moment \( M_0 \) is equal to the design axial force times the minimum eccentricity. If the wall was designed as a column the minimum eccentricity would be \( 0.05t_w \) and Eqn 30 becomes:

\[
M_{xx}^* = \delta_b N_x 0.00625t_w \pi^2 \sqrt{\frac{0.4E_{sec}}{E_c}} \left\{ 1 + \nu \right\}
\]

\[
M_{yy}^* = \delta_b N_x 0.0025t_w \pi^2 \left\{ 1 + \nu \right\}
\]

(31)

If the upper limit on the amplified eccentricity in the \( x \) direction is \( t_w/6 \), then for the assumption of no tension to apply, the amplification factor should be restricted to

\[
\delta_b \leq \frac{42.16}{\pi^2(1 + \nu)} \sqrt{\frac{E_c}{E_{sec}}}
\]

(32)

When checking the design capacity of the wall, the design actions \( N_x^* \) and \( M_{xx}^* \) need to be compared to the wall ultimate capacity by way of a column interaction diagram. The design moment in the \( y \) direction \( M_{yy}^* \) must be less than the yield moment. Note that the minimum steel required for high strength concrete members is higher than specified in AS3600. For moderate crack control the minimum percentage of tensile steel \( p_{min} \) in AS3600 is given by \( 1.0/ f_y \) where \( f_y \) is the yield stress of the tensile reinforcement. To calculate an approximate formula for the minimum percentage tensile steel, the condition that the ultimate moment \( M_u \) must be greater than 1.2 times the cracking moment \( M_{cr} \) can be used. For a rectangular section \( b \) wide and effective depth \( d \) with tensile steel of area \( A_{st} \) we can write
\[ M_{u} > 1.2 M_{cr} \]

\[ A_{st} f_{sy} 0.9d > 1.2 \times 0.6 \sqrt{f_{c}'} \frac{b(1.1d)^{2}}{6} \]

\[ p_{\text{min}} > 0.165 \frac{\sqrt{f_{c}'}}{f_{sy}} \quad (SI \text{ Units}) \] (33)

Figure 10 compares the above formula with the code requirement for walls. It is seen that the code requirement is too small for high strength concrete. The yield moment capacity per unit width for a rectangular section is given by

\[ \phi M_{\text{yield}} = \phi A_{st} f_{sy} d \left( 1 - \frac{1}{3} k \right) \]

with \( k = \sqrt{(np)^{2} + 2np - np} \)  
\[ n = \frac{E_{s}}{E_{c}} \quad p = \frac{A_{st}}{bd} \] (34)

where \( E_{s} \) is the elastic modulus for steel. The strength reduction factor \( \phi \), for pure bending is 0.8. If minimum steel is used according to Eqn 33 and \( E_{c} \) is set to 5056\( \sqrt{f_{c}'} \) and 400 MPa grade reinforcement is used, Eqn 34 simplifies to

\[ \phi M_{\text{yield}} = \phi bd^{2} 0.156 \sqrt{f_{c}'} \quad (SI \text{ Units}) \] (35)

5. EXAMPLE OF A SLENDER HIGH STRENGTH REINFORCED CONCRETE CORE WALL

Consider a diaphragm type wall in a multistorey building where \( b = 8000 \) mm, the thickness is 300 mm and \( a \gg b \). The concrete strength is 100 MPa. The loading is predominantly due to gravity type loads with a design axial force of 4500 N/mm. Let's assume for this example that the wall can be considered to be simply supported on all four sides (other boundary conditions could be examined following the same procedure). The slenderness of this wall is \( 8000/300 = 27 \). From Fig. 9 the buckling load is
\[ N_{cr} = 0.81 f'c t_w = 0.81 \times 100 \times 300 = 24300 \, N/mm \]

Since the majority of the loading is gravity type sustained load let \( \beta_d \) be one. The amplification factor is therefore

\[ \delta_b = \frac{1}{1 - \frac{2 \times 4500}{24300}} = 1.59 \]

The stress ratio \( f/f'c \) is 4500/(100x300) = 0.15 and therefore from Fig. 8 the secant modulus is equal to the elastic modulus. The upper limit on the amplification factor is therefore

\[ \delta_b \leq \frac{42.16}{\pi^2 (1 + \nu)} = 3.56 \quad : \quad OK! \]

The design amplified moments are therefore

\[ M_{xy}^* = 1.59 \times 4500 \times 0.05 \times 300 \times \frac{\pi^2}{8} \times 1.2 \times \sqrt{0.4} \]
\[ = 100 \, kNm/m \]

\[ M_{yy}^* = 1.59 \times 4500 \times 0.02 \times 300 \times \frac{\pi^2}{8} \times 1.2 \]
\[ = 64 \, kNm/m \]

The load eccentricity being 100000/4500 = 22.3 mm = 0.07 \( t_w \). Inspection of the wall interaction diagram ignoring any reinforcement showed that the design actions \( N_{cr}^* \) and \( M_{xx}^* \) were well within the safe zone. The \( M_{yy}^* \) should not exceed the yield moment capacity and hence reinforcement should be provided to meet this requirement. The minimum steel according to Eqn 34 using 400 MPa steel is \( p_{min} = 0.004125 \). The yield capacity with minimum steel based on Eqn 35 is

\[ \phi M_{yield} = 0.8 \times (0.9 \times 300)^2 \times 0.156 \sqrt{100} \]
\[ = 91 \, kNm/m \quad > \quad 64 \, kNm/m \]
The wall thickness could be reduced keeping in mind that the reinforcement in the x direction (vertical steel) has been ignored and the distribution of axial loading through the building has also been ignored. Table 1 contains further calculations for the amplified moments with various wall thickness. The buckling load estimate would increase if the long edges are considered fully fixed and this would further reduce the amplification factor.

6. CONCLUSIONS

The present design methods for reinforced concrete walls are generally for walls in one-way bending. Core walls in multi-storey structures have boundary conditions such that any bending induced by the presence of imperfections or load eccentricity will involve two-way bending. The application of design formulae for walls in one-way bending to walls in two-way bending are very conservative. As with columns in the lower storeys, economic benefits can be gained by reducing the core wall thickness through the use of high strength concrete. Reducing the wall thickness however, results in walls of greater slenderness which require an assessment of the amplification effects of buckling. The limit on the slenderness ratio of walls in most codes is generally 30. To take full advantage of the use of high strength concrete, a more detailed assessment of the effects of buckling are required incorporating the two-way nature of bending in core walls.

This paper has presented an approximate analysis for the buckling load for simply supported reinforced concrete walls. The buckling load is then used to estimate the amplified moments in the longitudinal and transverse directions. The derivations have assumed that the axial load eccentricity can be taken as 0.05 times the wall thickness as is used for columns in AS3600. This is different to what is assumed for walls in one-way bending where the initial eccentricity is taken as 0.167 times the wall thickness. An upper limit on the amplification factor is determined. Formulae for the minimum percentage reinforcement and yield moment capacity for bending in the transverse direction are derived. It is found that the critical direction for capacity is not necessarily in the direction of loading since the moment capacity is increased due to the presence of the axial load.
Table 1: Amplified Moments versus Wall Thickness

<table>
<thead>
<tr>
<th>t_w (mm)</th>
<th>300</th>
<th>275</th>
<th>250</th>
<th>225</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>b/ t_w</td>
<td>27</td>
<td>29</td>
<td>32</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td>δ_b</td>
<td>1.59</td>
<td>1.72</td>
<td>1.92</td>
<td>2.6</td>
<td>5.5</td>
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<td>Mxx (kNm/m)</td>
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<td>100</td>
<td>101</td>
<td>123</td>
<td>232</td>
</tr>
<tr>
<td>Myy (kNm/m)</td>
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<td>63</td>
<td>64</td>
<td>78</td>
<td>147</td>
</tr>
<tr>
<td>δ_Myield</td>
<td>91</td>
<td>76.5</td>
<td>63.2</td>
<td>51</td>
<td>40</td>
</tr>
</tbody>
</table>

7. REFERENCES

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