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STABILITY OF TAPERED I - BEAMS

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STABILITY OF TAPERED I-BEAMS

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7. Abstract

The British and Australian limit states design rules for the lateral buckling limit state of tapered I-beams are reviewed. A finite element method is described, and this is used to develop solutions for the elastic critical loads of beams which cover an extensive range of geometries and loading conditions. A design method is proposed which makes use of the accurate elastic critical solutions. The code methods and the accurate proposal are compared by an example.

8. Number of pages 12 pages plus figures

1. *INTRODUCTION*

Tapered I-beams fabricated by welding, such as that shown in Fig. 1, have become a viable alternative to uniform beams because of the reduced costs of fabricating plated steel members. The advantage of using a tapered beam instead of a uniform beam is that the member may be used in situations where the major axis bending moment varies along the length of the beam, so that economy can be gained by reducing the member section in the regions of low bending moment. Non-uniform I-beams may be tapered in their depth, or in their flange width, but rarely in their flange or web thicknesses.

If a tapered beam does not have sufficient lateral stiffness or lateral support to allow its cross-sectional strength to be reached (which for compact sections is the full plastic moment), then the strength of the beam is governed by its resistance to flexural-torsional buckling. However, significant economies in steel can still be achieved if the elastic critical load can be determined for the tapered beam. This paper is concerned with design against such instability of tapered I-beams.

A detailed review of research on the lateral stability of tapered I-beams prior to the early 1970's has been given by Kitipornchai and Trahair [1]. Contributions in the field since then have included those by Lee, Morrell and Ketter [2], Nethercot [3], Prawel, Morrell and Lee [4], Horne, Shakir-Khalil and Akhtar [5], Salter, Anderson and May [6], Brown [7] and Shiomi and Kurata [8]. However, while several other papers on the lateral stability of tapered beams may be cited as well, there have been few general approaches to the problem contributed [9].

In this paper, the proposals of the British and Australian limit states design (LSD) codes for the instability limit state of tapered I-beams are briefly reviewed. A general finite element method [10], suited to microcomputer applications, is then summarised. Parametric solutions for the elastic lateral buckling of tapered beams, determined from the finite element program, are then given, and these are presented as an accurate alternative to the proposals in the LSD codes. Finally, a design proposal is presented, and this is illustrated by an example.

2. *LSD CODE RULES*

In the British BS5950 [11], tapered doubly symmetric I-beams are designed by a modification of the rules for prismatic members. The elastic critical moment M_E is used as a basis, and is calculated from

$$M_E = \frac{M_P \pi^2 E}{\lambda_{LT}^2 F_Y} \tag{1}$$

in which M_P is the full plastic moment of the section at the point where the factored applied moment is greatest, and where λ_{LT} is the "equivalent slenderness". In the BS5950, the equivalent slenderness is

calculated by modifying the beam slenderness $\lambda = l/r$, where r is the radius of gyration at the point of maximum applied moment and l is the effective length, by

$$\lambda_{LT} = nv\lambda \tag{2}$$

In Eq. 2, n is a coefficient related to the degree of tapering, given by

$$n = 1.5 - 0.5R_f > 1.0 \tag{3}$$

where R_f is the ratio of the flange area at the point of minimum moment to that at the point of maximum moment, and is always equal to unity when the flange does not taper. The coefficient v in Eq. 2 is a slenderness factor, which is related to λ and to the torsional index x by

$$v = [1 + (\lambda/x)^2/20]^{-1/4} \tag{4}$$

where

$$x = 0.566d\sqrt{A/J} \approx D/T \tag{5}$$

in which D is the overall beam depth, and where A and J are the area and torsion constant of the member at the point of maximum moment respectively.

Finally, the elastic buckling moment M_E is related to the design strength M_b of the tapered beam by use of the Perry equation [12]

$$M_b = \frac{M_E M_P}{\phi_B + \sqrt{\phi_B^2 - M_E M_P}} \tag{6}$$

where

$$\phi_B = \frac{M_P + (\eta_{LT}+1)M_E}{2} \tag{7}$$

in which the Perry coefficient η_{LT} for fabricated sections is given by

$$\eta_{LT} = 0.0056\sqrt{\pi^2 E/F_Y} \tag{8}$$

However, calculation of the resistance M_b of tapered beams in the BS5950 is not as difficult as Eqs. 1 to 8 would suggest, since most of these relationships are tabulated. In fact, for a given slenderness ratio λ, only the quantities λ/x (Eq. 5) and n (Eq. 3) need to be calculated when the tables in BS5950 are used.

The draft Australian AS1250 [13] provides a somewhat more accurate rule than that of the BS5950 to account for the effects of section tapering in determining the design strength M_b. This is again based on the elastic critical moment M_E, which is expressed as

$$M_E = \alpha_{st} M_o \tag{9}$$

where

$$M_o = \sqrt{(\pi^2 EI_y/l^2)\,(GJ+\pi^2 EI_\omega/l^2)} \tag{10}$$

is the elastic critical load of a prismatic beam of effective length l, and where

$$\alpha_{st} = 1.0 - 0.6\{1-(0.6+0.4\frac{D_m}{D_c})\frac{A_m}{A_c}\} \tag{11}$$

In this equation, A_m, A_c are the flange areas and D_m, D_c are the section depths at the minimum section, and at the critical section where the ratio of the bending moment to the full plastic moment is greatest. Equation 11 was designed to be an approximation to the limited results of Kitipornchai and Trahair [1], and its basis is illustrated in Fig. 2 [14]. The minor axis second moment of area I_y, torsion constant J and warping constant I_ω in Eq. 10 should also be determined from the section properties at the critical section. The effect of α_{st} in reducing the elastic critical moment in Eq. 9 is similar to the reduction $1/n^2$ in the elastic critical moment afforded by the BS5950.

The design strength M_b is then obtained from M_E as

$$M_b = \alpha_m \alpha_s M_P \tag{12}$$

where the slenderness reduction factor α_s is given by

$$\alpha_s = 0.6\{[(M_P/M_E)^2+3]^{1/2} - M_P/M_E\} \tag{13}$$

and is analogous to Eqs. 6 to 8 above of the BS5950. The moment distribution factor α_m in Eq. 12 is tabulated in the AS1250, and may conservatively be taken as unity, that latter approximation being the same as in the BS5950.

The AS1250 also permits "design by buckling analysis" [12], in which Eqs. 12 and 13 are used, with the elastic critical moment M_E replaced by

$$M_E = M_{ob}/\alpha_m \tag{14}$$

where M_{ob} is the elastic critical moment determined using the results of an elastic buckling analysis that takes account of the support, restraint and loading conditions, and of the tapering of the member. The results of such an analysis are given in Section 4 of this paper, and are discussed subsequently. The moment modification factor α_m may again be taken as unity in design by buckling analysis, or as the values tabulated in AS1250.

The use of Eqs. 12 to 14 for tapered members implies that the interaction between elastic buckling and yielding that determines the strength of non-prismatic members is the same as the interaction that governs the strength of prismatic members. In the absense of sufficient relevant tests, the validity of these equations for tapered beams was investigated herein by comparing the predictions of design by buckling analysis to AS1250 with the results of a materially and geometrically nonlinear analysis of web tapered beams with a moment at one end undertaken by Shiomi and Kurata [8]. These results are compared in Fig. 3 for web taper constants α_w in the range 0.4 to 0.7. The predictions of Eqs. 12

to 14 in Fig. 3 are for $\alpha_m = 1.0$ and $\alpha_m = 1.75$ (which is the approximation for the loading considered [12]), with $M_{ob(l)}$ and $M_{P(l)}$ being the values of M_{ob} and M_P at the largest section. It can be seen that the use of Eq. 13 with $\alpha_m = 1.0$ is a conservative lower bound prediction of Shiomi and Kurata's results, while the use of this equation with $\alpha_m = 1.75$ is a conservative lower bound prediction of the mean results. Because the strengths to AS1250 are reduced in design by a capacity reduction factor of 0.9, which accounts in part for the scatter of the accurate results [15], it appears that the method of design by buckling analysis in the AS1250 is suitable for tapered beams, provided that the elastic critical moment can be determined accurately.

3. *FINITE ELEMENT BUCKLING ANALYSIS*

The use of finite elements to solve lateral buckling problems dates back to the work of Barsoum and Gallagher [16] and Powell and Klinger [17] in 1970. The one dimensional line elements developed by these researchers assumed the coincidence of the axis of twist with the shear centre axis which is parallel to the centroidal axis. Uniform elements similar to these have been used by Nethercot [3] to study the flexural-torsional buckling of tapered beams.

However, application of uniform elements to approximate a tapered beam causes difficulties because of the artificial discontinuities introduced at the centroidal and shear centre axes at nodes. In addition, the rate of convergence is very slow [10] because of the comparatively crude model provided for representing a tapered element. In order to overcome these difficulties, a one dimensional finite element has been developed [10] to provide an accurate and rapidly converging method of representing a tapered member, and which does not introduce any artificial discontinuities. This element has been validated by comparisons with more complex, but less general, numerical treatments.

The finite element method described in Ref. 10 is superior to the use of uniform elements, in that it correctly caters for the effects of non-uniformity. This is achieved by abandoning the usual shear centre and centroidal axis systems in the development of the line element. The element uses a convenient and arbitrary Cartesian axis system passing through the mid-height of the web as the reference axis for lateral displacements and twists. The stiffness and stability matrices are easily calculated by making this assumption of an arbitrary axis of twist.

The convergence of the finite element analysis which uses tapered elements has been demonstrated in Ref. 10, where it was shown that very few elements are required to obtain an accurate solution. Because of this, the global banded stiffness and stability matrices are of modest size, and a microcomputer may be used to achieve a rapid solution of the problem. The eigenvalue routines given by Hancock [18] are suitable for extracting the buckling load and mode from these global matrices, and were employed in the study.

4. *PARAMETRIC STUDY*

4.1 *General*

The finite element method [10] discussed in the previous section has been used to calculate the elastic critical loads or moments of tapered doubly symmetric I-beams for use in design. Solutions are given for a beam with flange or web taper with concentrated end moments, and for a beam with flange or web taper acted upon by a uniformly distributed load.

The differential equations for buckling derived by Kitipornchai and Trahair [1] indicate that the beam parameter K is an independent variable, where

$$K = \frac{\pi}{L}\sqrt{EI_\omega/GJ}_{(1)} \tag{15}$$

in which L is the length of the beam, and the subscript (1) indicates the geometry at the largest section. Because of this, only one section with d=200mm, B=50mm, T=3mm and t=2mm was used to calculate K for the parameter study. Other geometries and beam lengths that gave the same value of K were tried as well, and it was found that these had negligible effect on the buckling solution, indicating the accuracy of the inference of Kitipornchai and Trahair's equations. The value of K may be interpreted as a measure of the slenderness of the beam, with small values of K indicating slender beams, while stocky beams are represented by large values of K.

4.2 *Beam with Moments at the Ends*

The lateral stability of the simply supported tapered beam shown in Fig. 1 has been studied, and plots of the dimensionless elastic critical moment

$$\gamma_M = M_{ob}L/\sqrt{EI_yGJ}_{(1)} \tag{16}$$

are given in Figs. 4a to 4e as functions of the moment gradient parameter β. In these figures, the solid lines are for $\alpha_f = 1$ with α_w varying, while the dashed lines are for $\alpha_w = 1$ with α_f varying. It can be seen from the figures that while the reductions in γ_M due to increasing flange taper are quite large, those due to increasing web taper are much less. Also of interest is the observation that for stocky beams the elastic critical moment is higher for the β=0.5 loading case than for the β=1.0 loading case (the latter being the safest loading condition for uniform beams [12]), and that this trend increases as the taper constants α_f and α_w decrease.

The use of Figs. 4a to 4e represents an accurate alternative to the codified design approaches for tapered beams with end moments, since many more parameters are treated than in Eqs. 3 and 11.

4.3 *Beam with Uniformly Distributed Load*

The lateral stability of the tapered beam loaded by a uniformly distributed load w shown in Fig. 5 has been studied. For this, the distributed load is assumed to act at a distance $\bar{a}$ below the web mid-height. For an isolated simply supported determinate beam, the end moment parameter β is zero, and values of the dimensionless elastic critical load

$$\gamma_w = wL^3/\sqrt{EI_y GJ_{(l)}} \tag{17}$$

are shown in the form of design curves in Figs. 6a to 6c as functions of K and the dimensionless load height parameter

$$\varepsilon = \frac{\bar{a}}{L}\sqrt{\frac{EI_y}{GJ}} \tag{18}$$

On the other hand, for "continuous" beams with the end moment parameter β being taken as unity, the corresponding plots of the dimensionless elastic critical load are shown in Figs. 7a to 7c. In both Figs. 6 and 7, the solid lines are for $\alpha_f = 1$ with α_w varying, while the dashed lines are for $\alpha_w = 1$ with α_f varying.

It can be seen from the figures that placing the load above the web mid-height (ε<0) results in a significant destabilising effect and reduces the buckling load, whilst placing the load below the web mid-height tends to stabilise the beam against lateral buckling. The reduction in γ_w below that for the corresponding uniform beam, expressed as a ratio, is shown in Fig. 8 for the beam with β=0 loaded at the centroid. The figure demonstrates that increasing the degree of flange taper reduces the ratio of the resistance of the tapered beam to that of the corresponding uniform beam. The reduction in the lateral buckling resistance for web tapered beams is less dramatic, however, with web tapering having little effect for the more slender beams. In all cases, the reductions in the lateral buckling resistance below that of the corresponding uniform beam increases as the beam parameter K increases and the beam becomes more stocky.

As for the previous sub-sections, the elastic solutions in Figs. 6 and 7 are based on a rational analysis, and are therefore of higher accuracy than the code approximations which are extrapolations from results of limited scope.

5. *DESIGN PROPOSAL*

The resistance of tapered beams may be calculated by the previously discussed method of design by buckling analysis from the accurate elastic buckling design curves presented in the previous section. The proposal advocated here is essentially that of the AS1250 LSD code, and is also applicable, with minor modification, to the design formulation of the BS5950.

Firstly, the design curves presented herein are used to calculate the elastic lateral buckling moment M_{ob} at the critical section, that is, the section where the ratio of the moment resulting from the factored load effects to the plastic moment is greatest. The critical moment M_{ob} includes the effects of off-shear centre loading and non-uniform moment distribution. Secondly, the elastic critical moment M_{obo} is obtained from the figures for shear centre loading and incorporating non-uniform moments, and the moment distribution factor α_m then calculated from

$$\alpha_m = M_{obo}/M_{oo} \tag{19}$$

where the elastic critical moment M_{oo} for uniform bending and centroidal loading is obtained from the design curves in Fig. 4a.

Finally, the design resistance M_b at the critical section is obtained from Eqs. 12 and 13, with α_m determined from Eq. 19 above and with M_E determined from Eq. 14. The philosophy of using this approach for relating the elastic critical moment to inelastic buckling and strength is discussed more fully in Ref. 12. The method may also be applied tentatively to design in accordance with the BS5950, with M_E determined as above and with Eq. 6 modified to

$$M_b = \frac{\alpha_m M_E M_P}{\phi_B + \sqrt{\phi_B^2 - M_E M_P}} \tag{20}$$

so as to conform with the strength curve in the BS5950, where ϕ_B and η_{LT} are given in Eqs. 7 and 8. The use of Eq. 20 is somewhat more rational than Eqs. 12 and 13 for fabricated tapered members, because it allows for an empirical adjustment of the Perry coefficient η_{LT} to make the theoretical predictions fit test results more closely.

6. *DESIGN EXAMPLE*

Problem

Calculate the bending resistance M_b of the tapered beam shown in Fig. 9 by
(i) the proposed design method;
(ii) the method of the AS1250 LSD code;
(iii) the method of BS5950.

Solution

(i) Assume the factored moment $M^* = 10000$kNm.

At the larger end: $M^*/M_P = 10000/3457 = 2.89$

At the smaller end: $M^*/M_P = 0.5\text{x}10000/753 = 5.18$

The smaller end is therefore critical.

$$K = \frac{\pi}{10000}\sqrt{\frac{200x10^3x6.430x10^{13}}{76.92x10^3x3.730x10^6}}$$

$$= 2.10.$$

From Fig. 4b, $\gamma_M = 6.9$.
At the larger end:

$$M_{ob} = \frac{6.9}{10000}\sqrt{200x10^3x1.786x10^8x76.92x10^3x3.730x10^6} \text{ Nmm}$$

$$= 2209 \text{ kNm}$$

Hence M_{ob} at the smaller end $= 0.5x2209 = 1104$ kNm.
From Fig. 4a, $\gamma_M = 5.0$, thus

$$\alpha_m = 6.9/5.0 = 1.38.$$

Using the AS1250 strength curve,

$$M_E = 1104/1.38 = 800 \text{ kNm}$$

$$\alpha_s = 0.6 \{ [(753/800)^2+3]^{1/2}-753/800 \} = 0.618$$

Hence

$$M_b = 1.38x0.618x753 = 642 \text{ kNm}.$$

Thus at the larger end,

$$M_b = 642/0.5 = 1284 \text{ kNm}.$$

Using the BS5950 strength curve,

$$\eta_{LT} = 0.0056\sqrt{\pi^2x200x10^3/275} = 0.474$$

$$\phi_B = \frac{753+(1+0.474)x800}{2} = 966 \text{ kNm}$$

Hence

$$M_b = \frac{1.38x800x753}{966+\sqrt{966^2-800x753}} = 539 \text{ kNm}.$$

Thus at the larger end,

$$M_b = 539/0.5 = 1079\text{kNm}.$$

This result is 6% lower than the Australian prediction (1284 kNm) based on the accurate curves. The reduction is due primarily to the different forms of Eqs. 6 and 13.

(ii) Since the minimum section is the critical section, $D_m = D_c$, $A_m = A_c$, so that $\alpha_{st} = 1.0$.

Hence

$$M_o = \sqrt{(\pi^2 x200x10^3 x1.786x10^8/10000^2)(76.92x10^3 x3.652x10^6) + \pi^2 x200x10^3 x4.019x10^{12}/10000^2)}\ \text{Nmm}$$

$= 1129$ kNm so that

$M_E = 1.0x1129 = 1129$ kNm and

$\alpha_s = 0.6 \{ [(753/1129)^2 + 3]^{1/2} - 753/1129 \} = 0.713$

For $\beta = -0.5$, AS1250 predicts $\alpha_m = 1.30$, so that

$M_b = 1.30x0.713x753 = 698$ kNm.

Thus at the larger end,

$M_b = 698/0.5 = 1396$kNm.

This result is 9% unconservative when compared with the accurate Australian solution (1284 kNm) based on the design curves.

(iii) Using the approximate British method,

$\lambda = 10000/85.3 = 117$

$n = 1.0$ since there is no flange tapering

$x = (1200+25)/25 = 49$

$v = [1+(117/49)^2/20]^{-1/4} = 0.940$

Hence

$\lambda_{LT} = 1.0x0.940x117 = 110$

$$M_E = \frac{3457x\pi^2 x200x10^3}{110^2 x275}\ \text{Nmm} = 2051\ \text{kNm}$$

$\eta_{LT} = 0.474$ as before

$$\phi_B = \frac{3457+(1+0.474)x2051}{2} = 3240\ \text{kNm}$$

$$M_b = \frac{2051x3457}{3240+\sqrt{3240^2-2051x3457}} = 1394\ \text{kNm.}$$

This result is 29% unconservative when compared with the accurate British solution (1079 kNm) based on the design curves.

7. *CONCLUSIONS*

The new Australian and British limit states steel codes provide for the buckling resistance of tapered I-beams fabricated by welding. These provisons are based on limited analyses of only a few geometrical and loading conditions, and are therefore approximate.

A finite element method of analysis suitable for studying the lateral buckling of tapered I-section beam-columns is briefly described. This method has been validated elsewhere, where it was shown to be accurate and to converge rapidly. The formulation is particularly suited to microcomputer applications.

The finite element method has been used to derive accurate elastic buckling resistances for tapered doubly-symmetric I-beams loaded by end moments or by a uniformly distributed load. A method of design is proposed, based on inelastic buckling, which transforms the accurate elastic solutions into member strengths. An example is given, and this demonstrates the inaccuracies of the LSD code approximations, particularly that of the BS5950. The example illustrates that little additional effort is required to use the accurate design curves, than is needed in present design to the Australian or British codes.

8. *ACKNOWLEDGEMENT*

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10. *NOTATION*

The geometrical parameters are shown in Fig. 1. Other principal notation is as below.

E,G	Elastic Young's modulus and shear modulus respectively;
F_Y	Yield stress;
I_y,I_ω	Minor axis second moment of area and warping constant respectively;
J	Torsion constant;

K	Beam parameter;
L	Beam length;
M_E	Modified elastic critical moment;
M_{ob}	Elastic critical moment of tapered beam;
M_{oo}	Critical moment in uniform bending;
M_P	Full plastic moment;
n	Taper coefficient in BS5950;
α_m	Moment modification factor;
α_s	Slenderness reduction factor;
α_{st}	Taper coefficient in AS1250;
β	Moment gradient parameter;
γ_M, γ_w	Dimensionless critical moments and loads respectively;
ε	Load height parameter;
η_{LT}	Perry coefficient in BS5950;
λ_{LT}	Equivalent slenderness in BS5950.

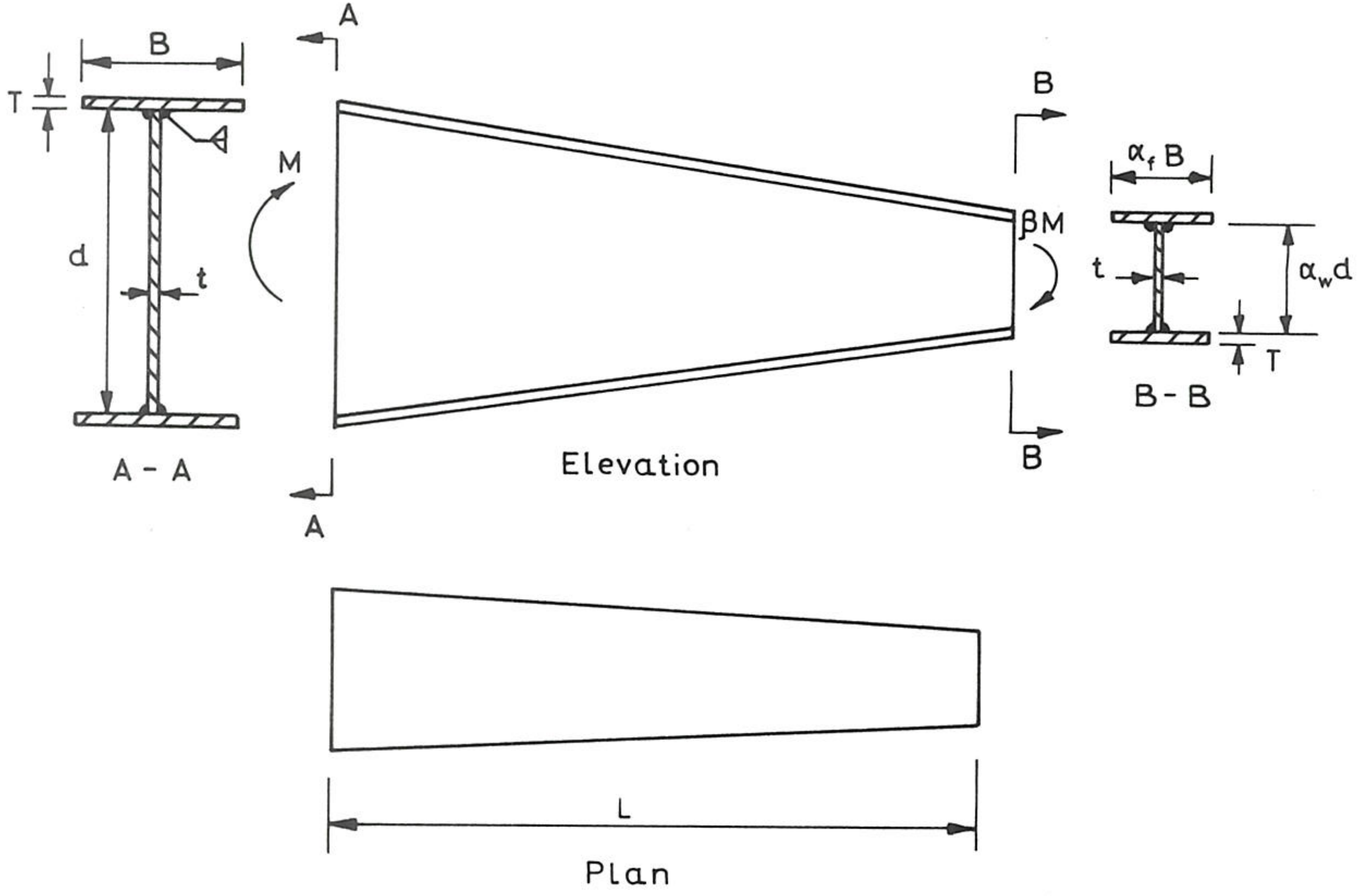


FIG. 1 DIMENSIONS OF TAPERED BEAM

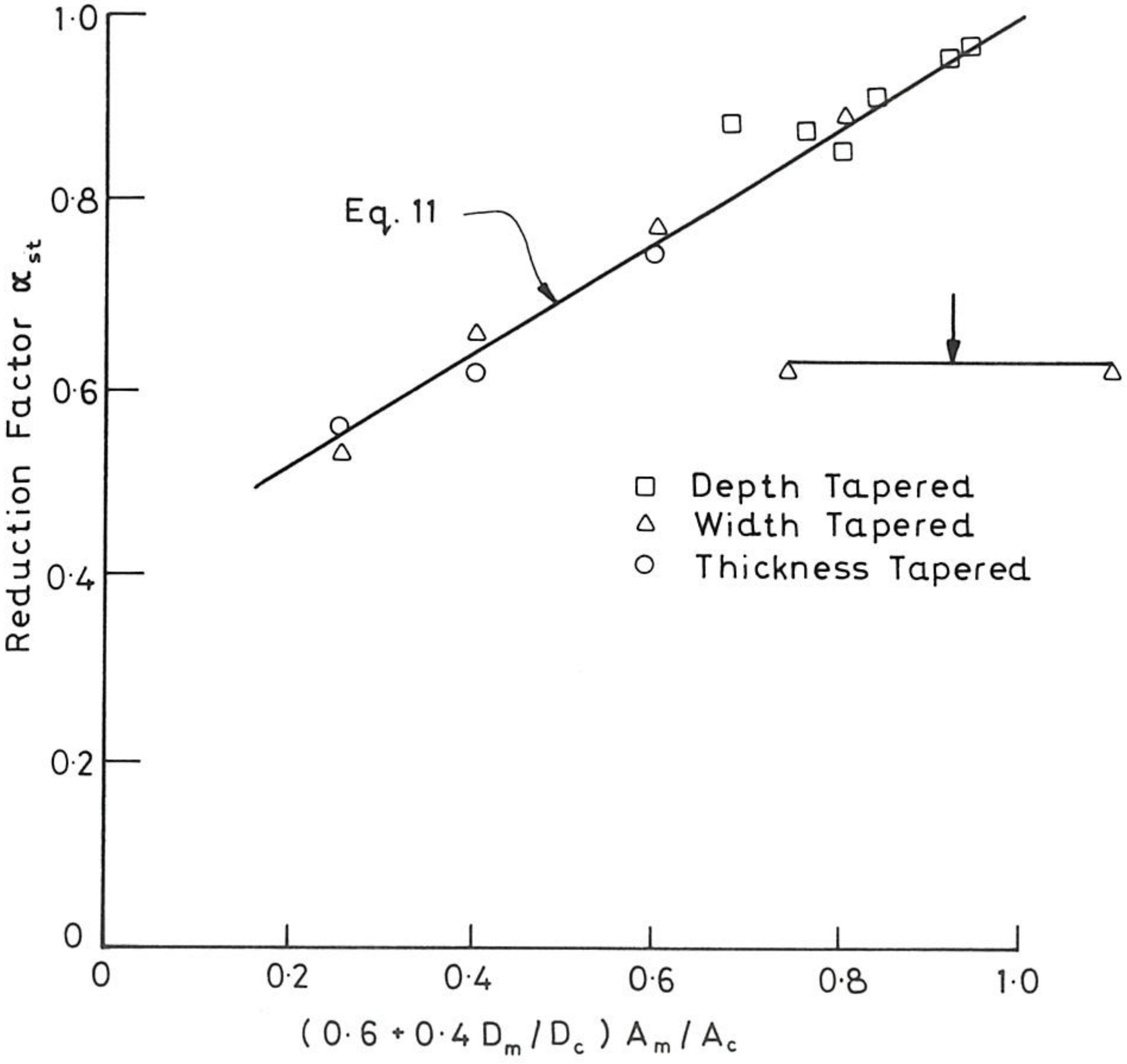


FIG. 2 BASIS FOR AS1250 RULE

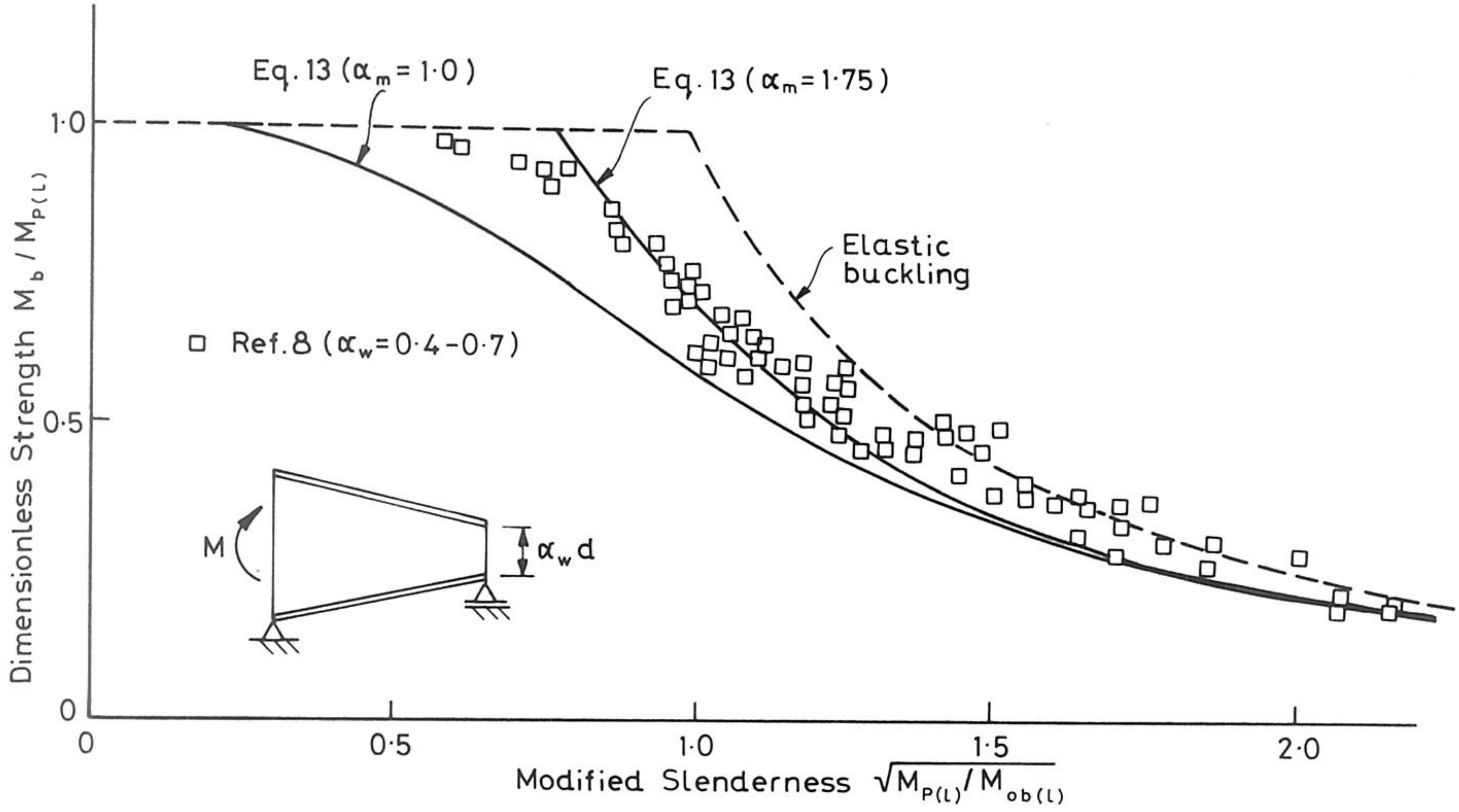


FIG. 3 STRENGTH PREDICTIONS FOR TAPERED BEAMS

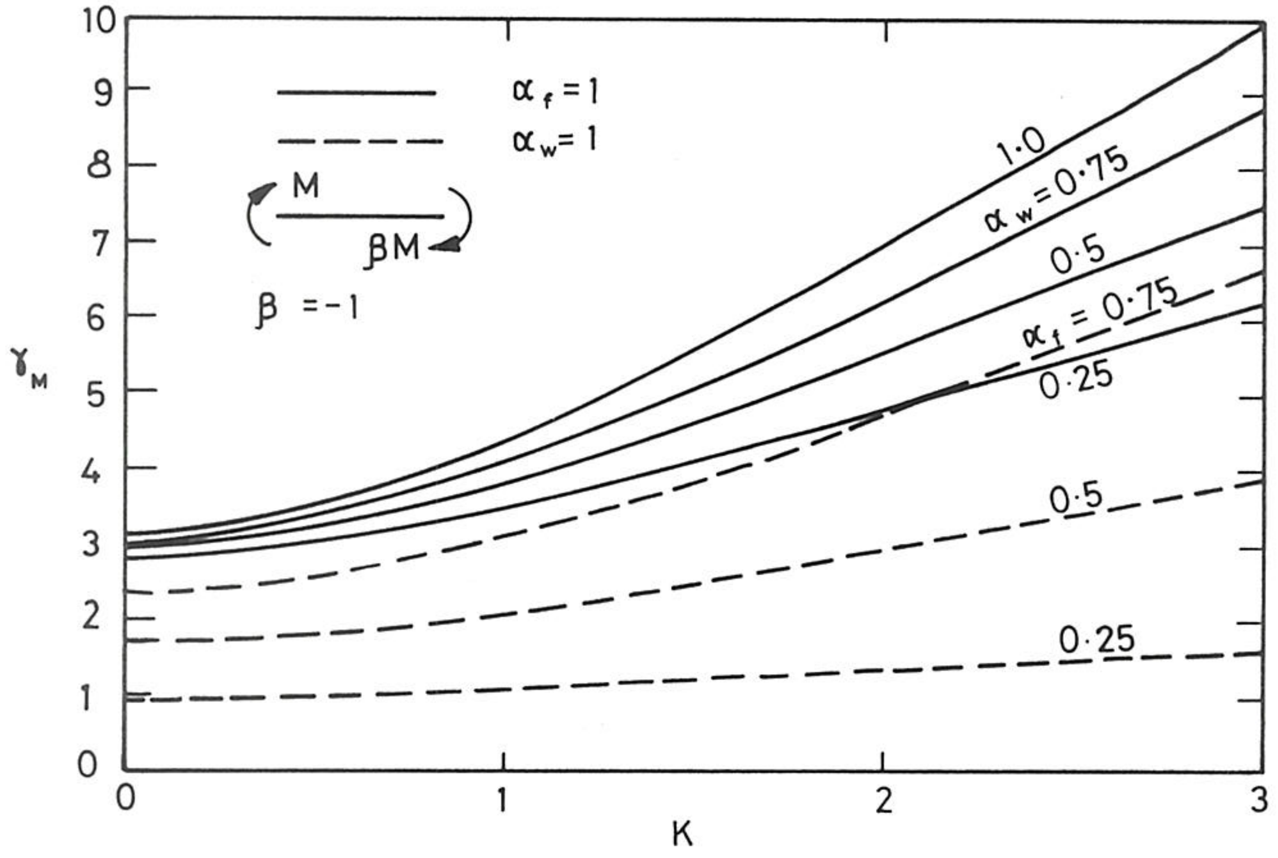


FIG. 4 (a) BEAM WITH END MOMENTS β = -1

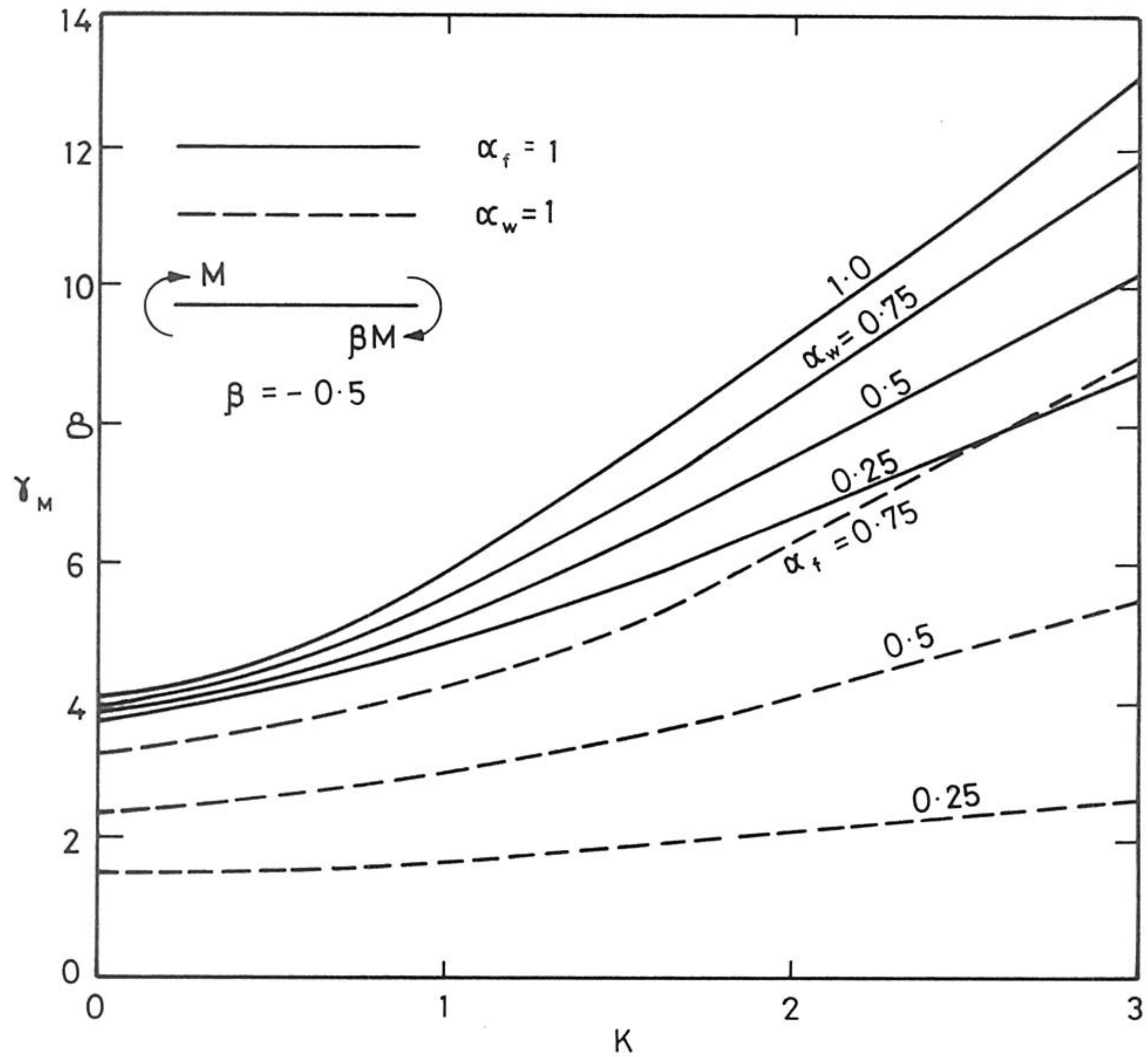


FIG. 4 (b) BEAM WITH END MOMENTS β = -0.5

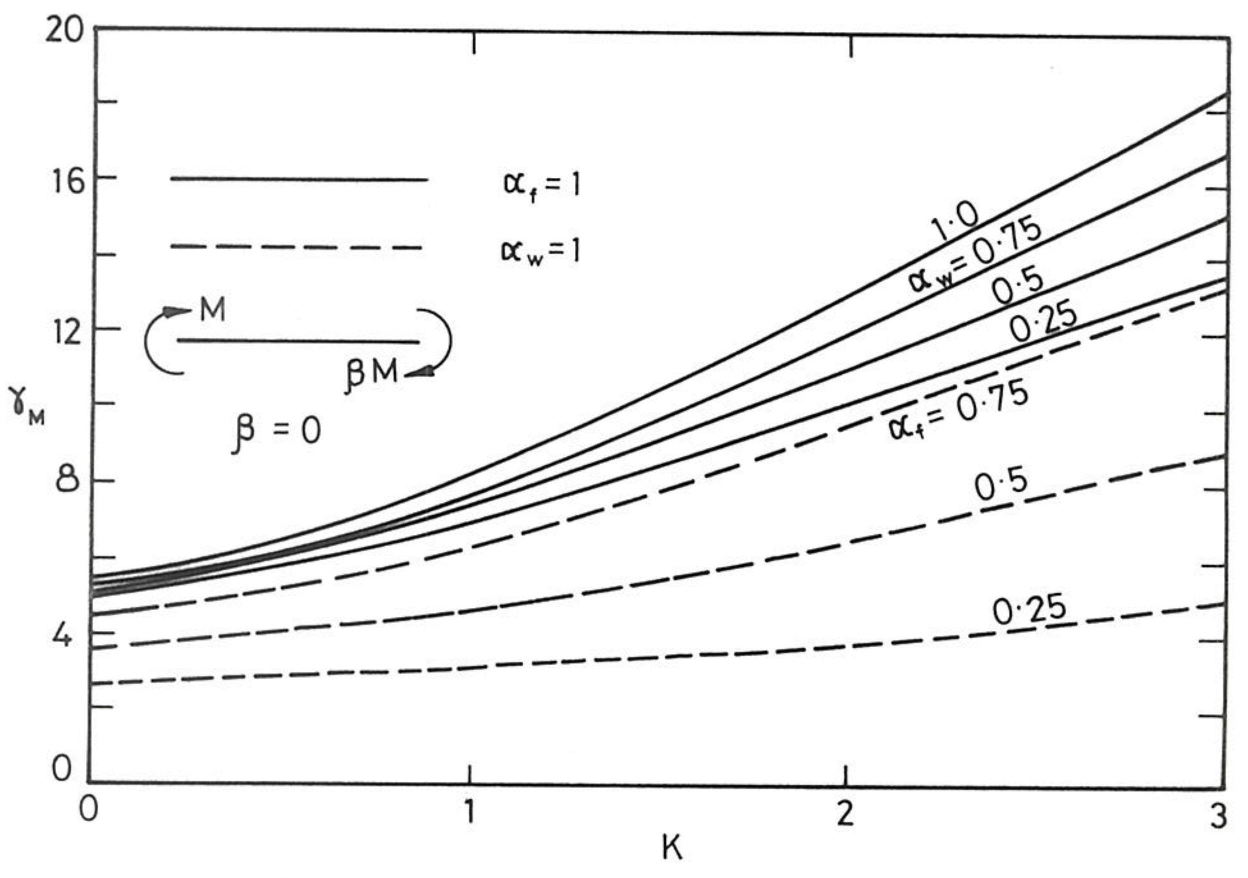


FIG. 4 (c) BEAM WITH END MOMENTS $\beta = 0$

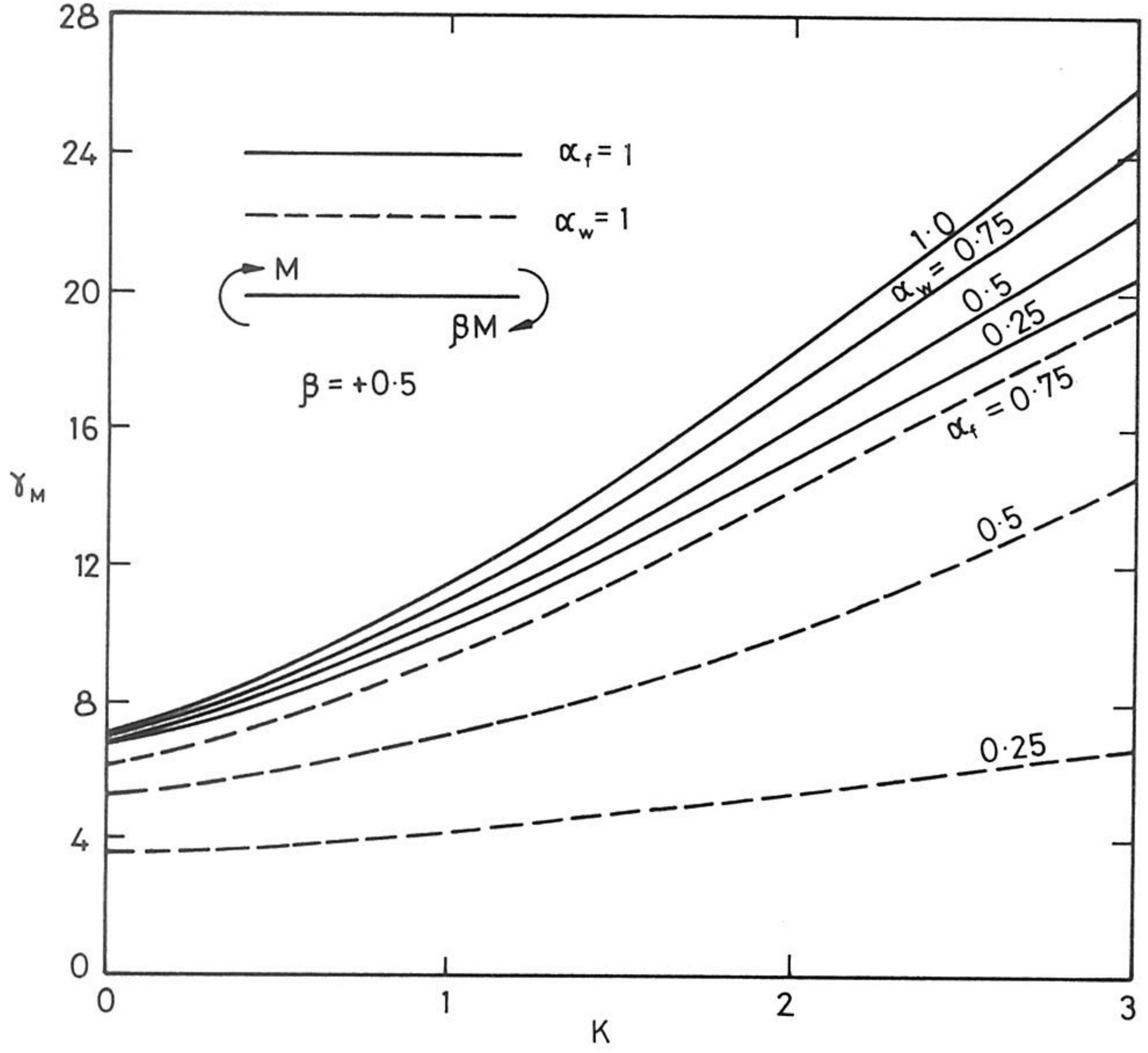


FIG. 4 (d) BEAM WITH END MOMENTS β = 0.5

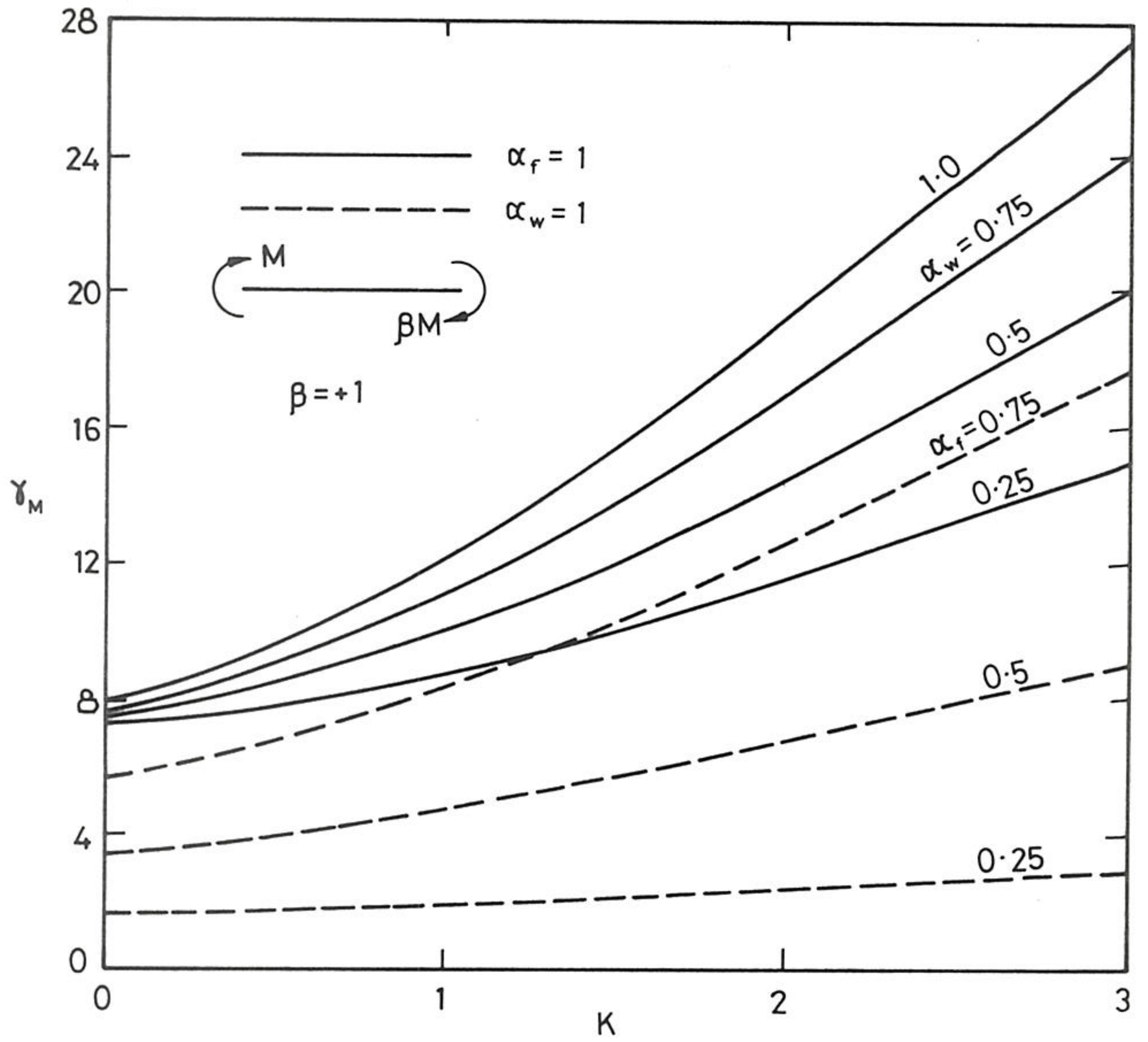


FIG. 4 (e) BEAM WITH END MOMENTS $\beta = 1$

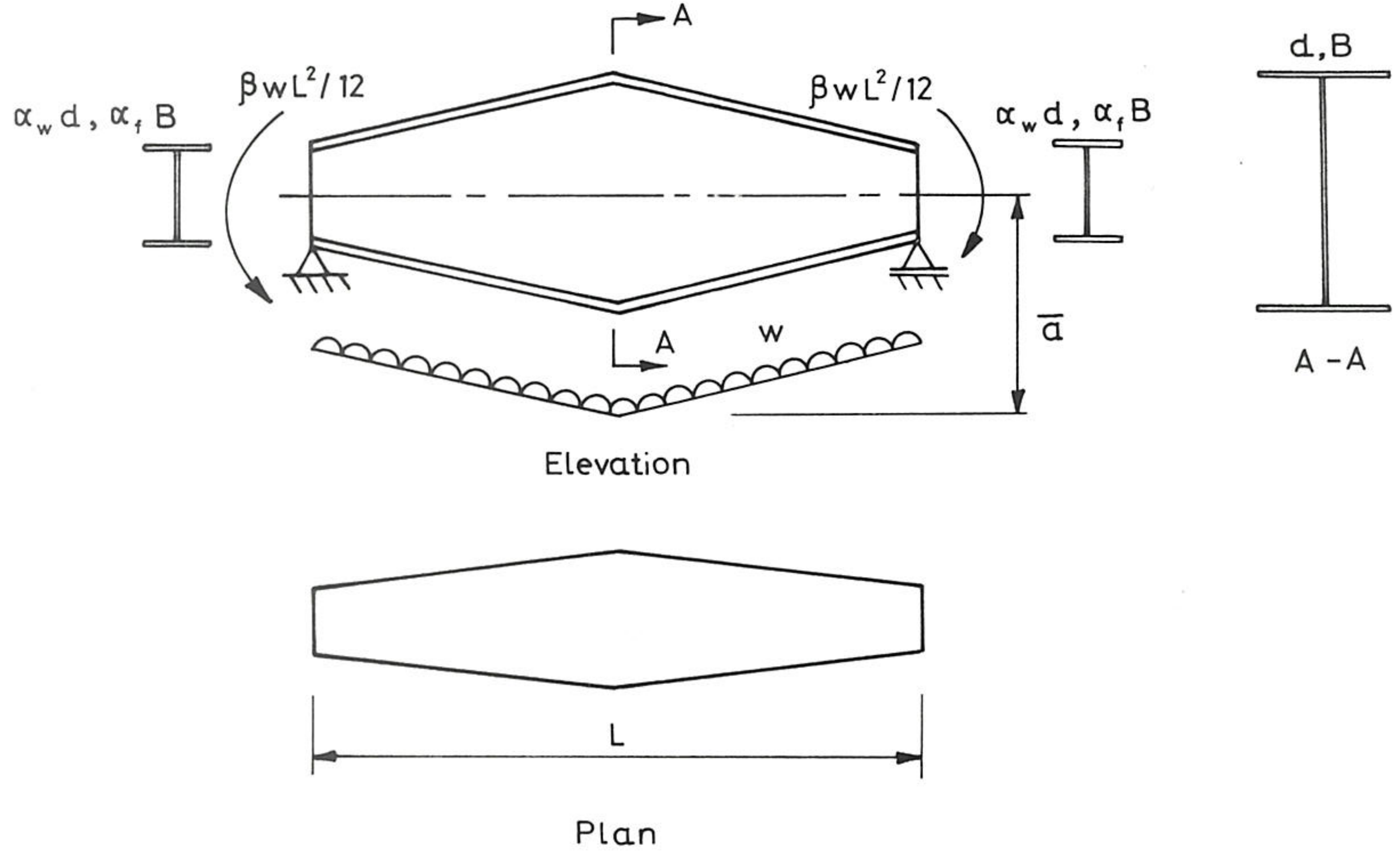


FIG. 5 TAPERED BEAM WITH U.D.L.

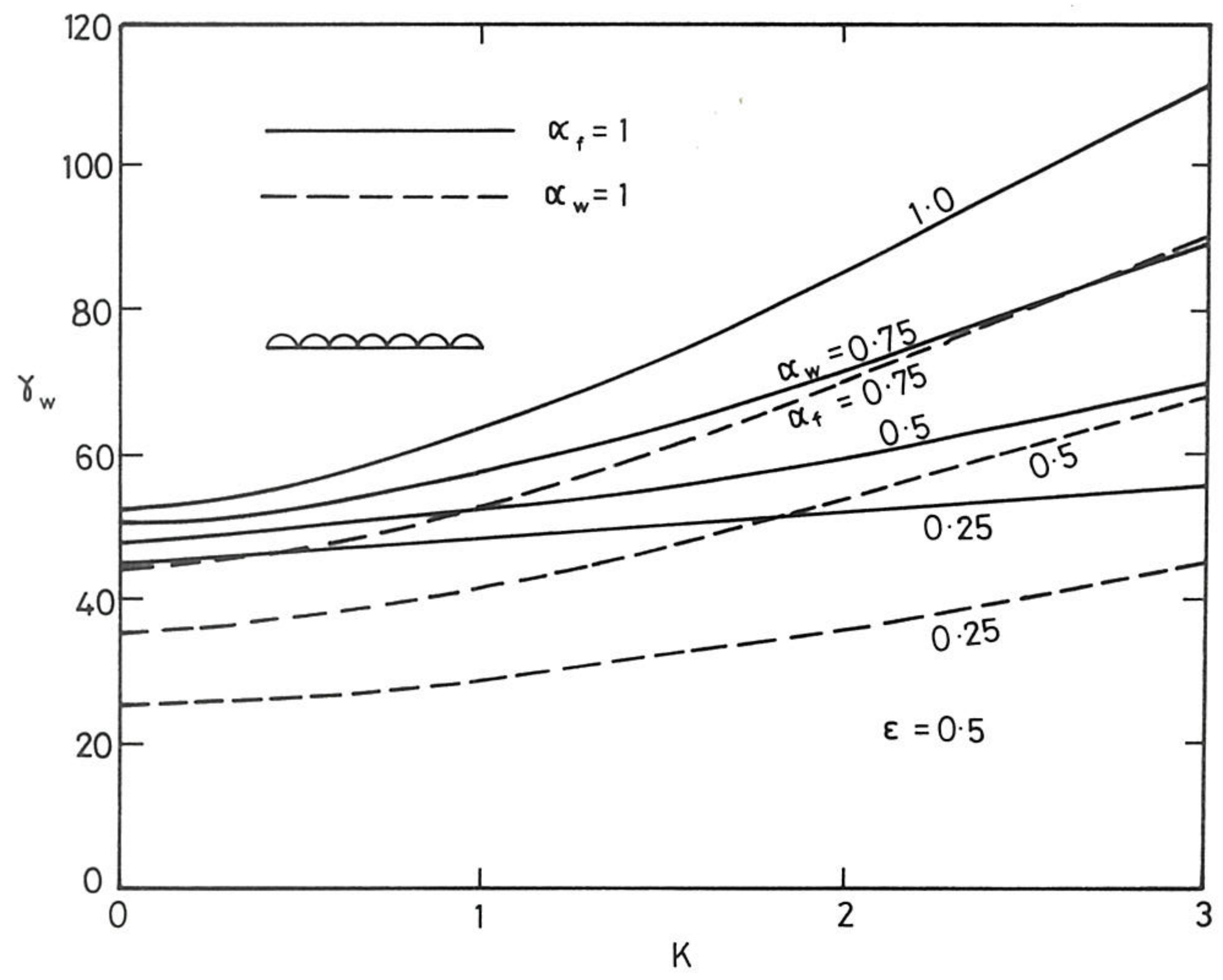


FIG. 6 (a) S.S. BEAM WITH U.D.L. $\varepsilon = 0.5$

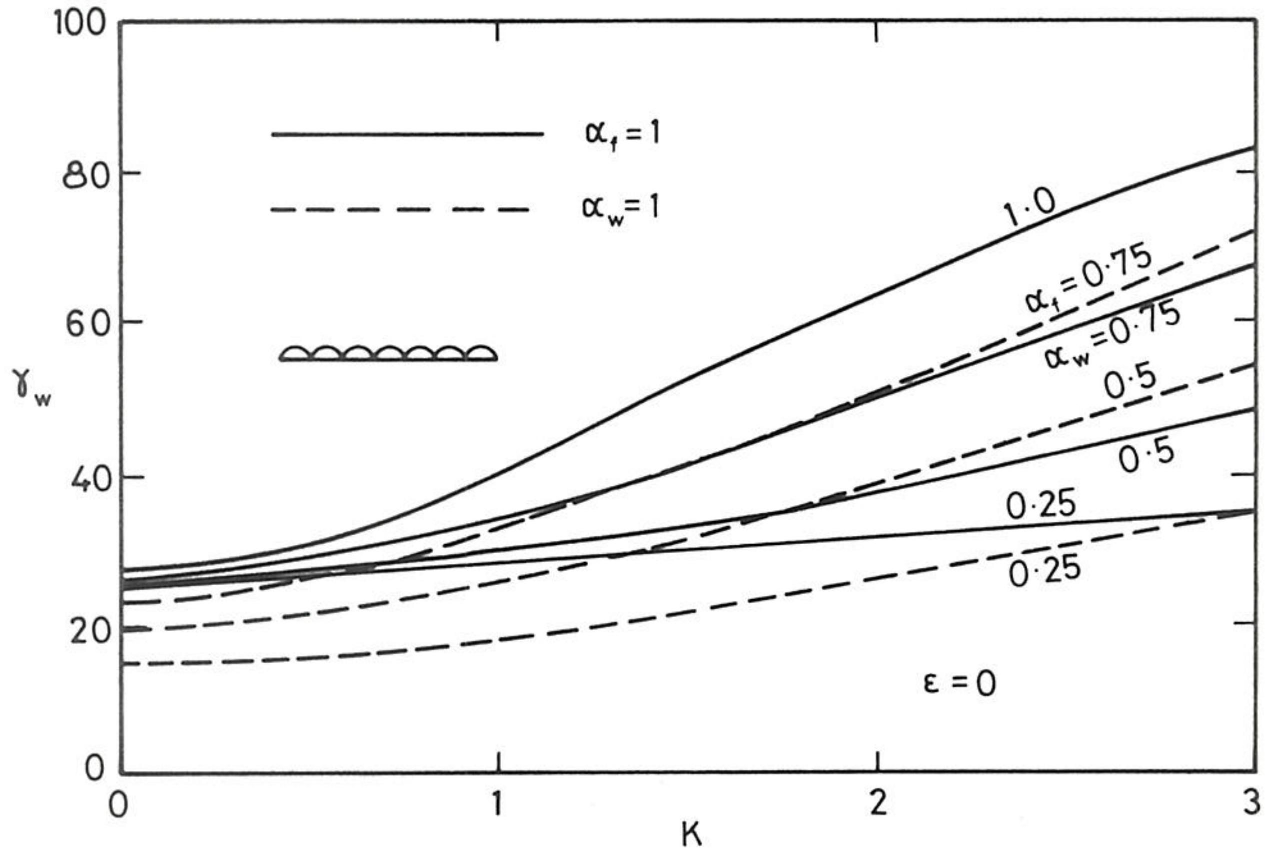


FIG. 6 (b) S.S. BEAM WITH U.D.L. $\varepsilon = 0$

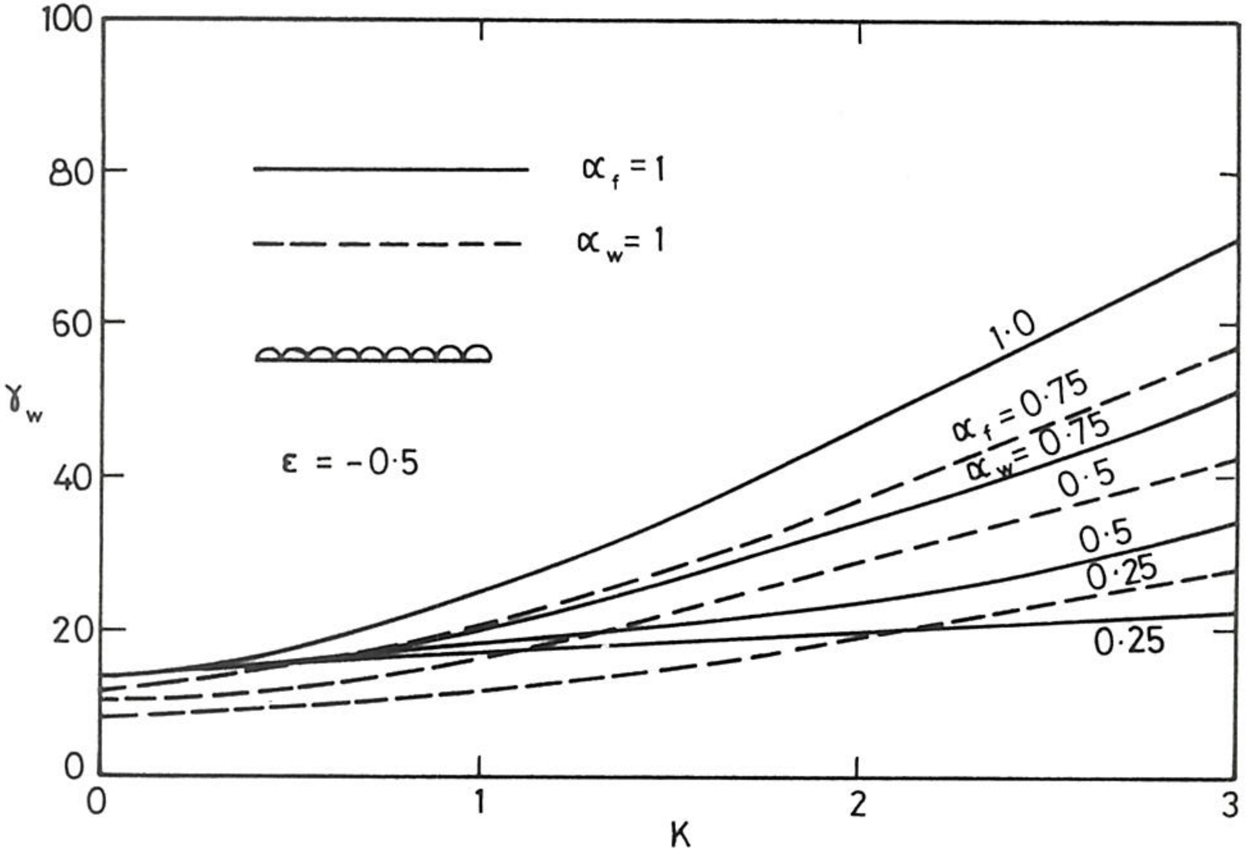


FIG. 6 (c) S.S. BEAM WITH U.D.L. $\varepsilon = -0.5$

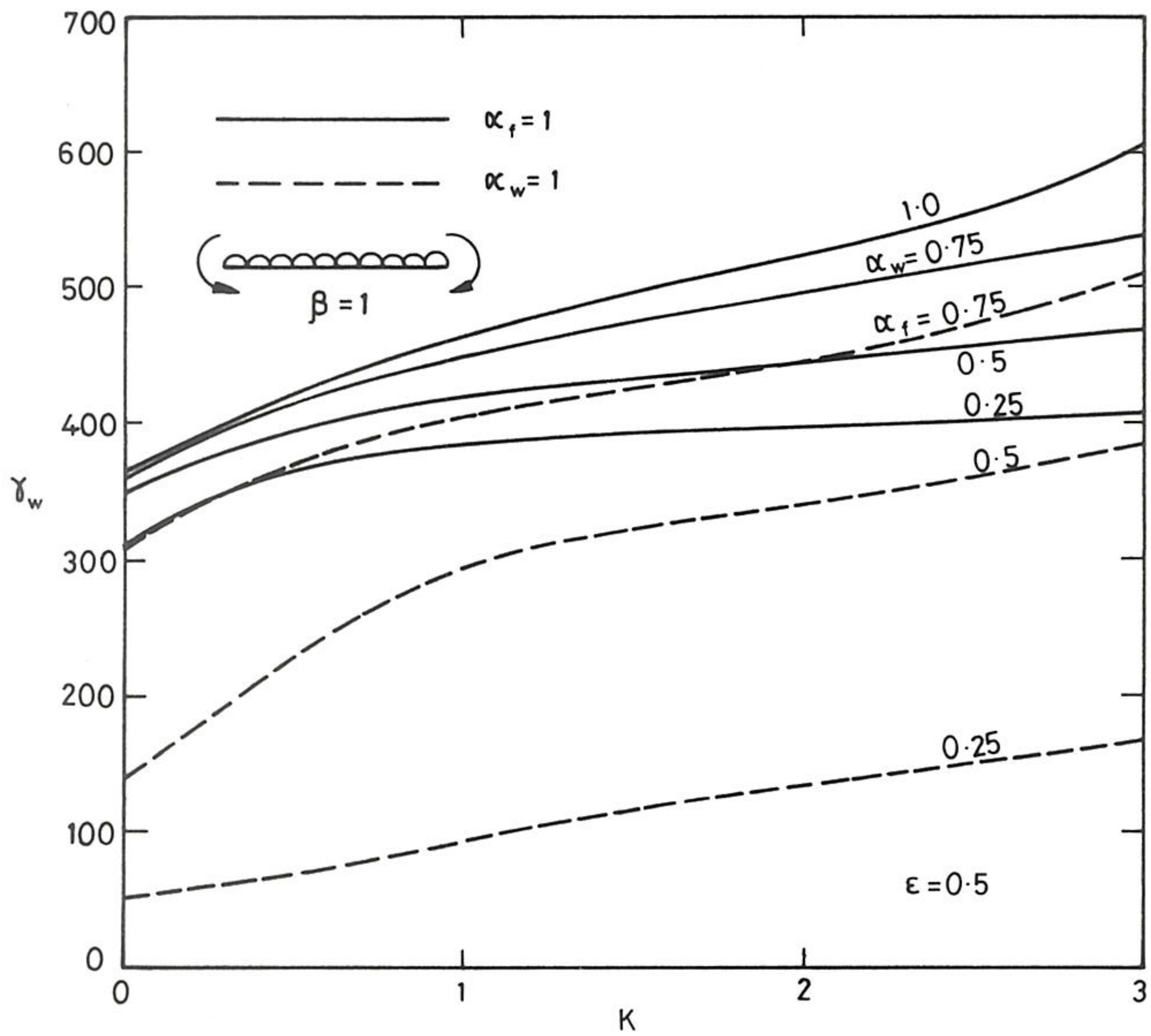


FIG. 7 (a) B.I. BEAM WITH U.D.L. ε = 0.5

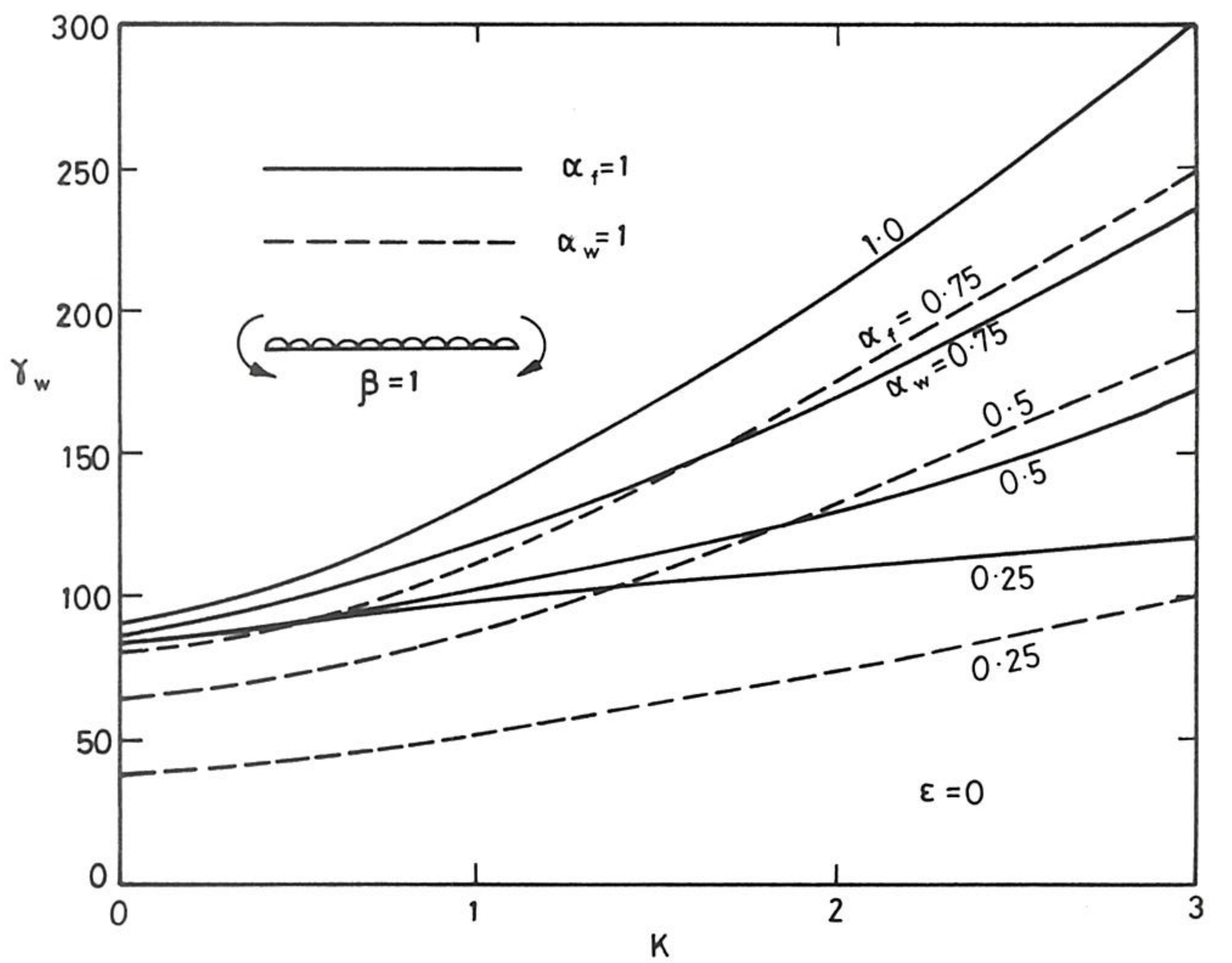


FIG. 7 (b) B.I. BEAM WITH U.D.L. $\varepsilon = 0$

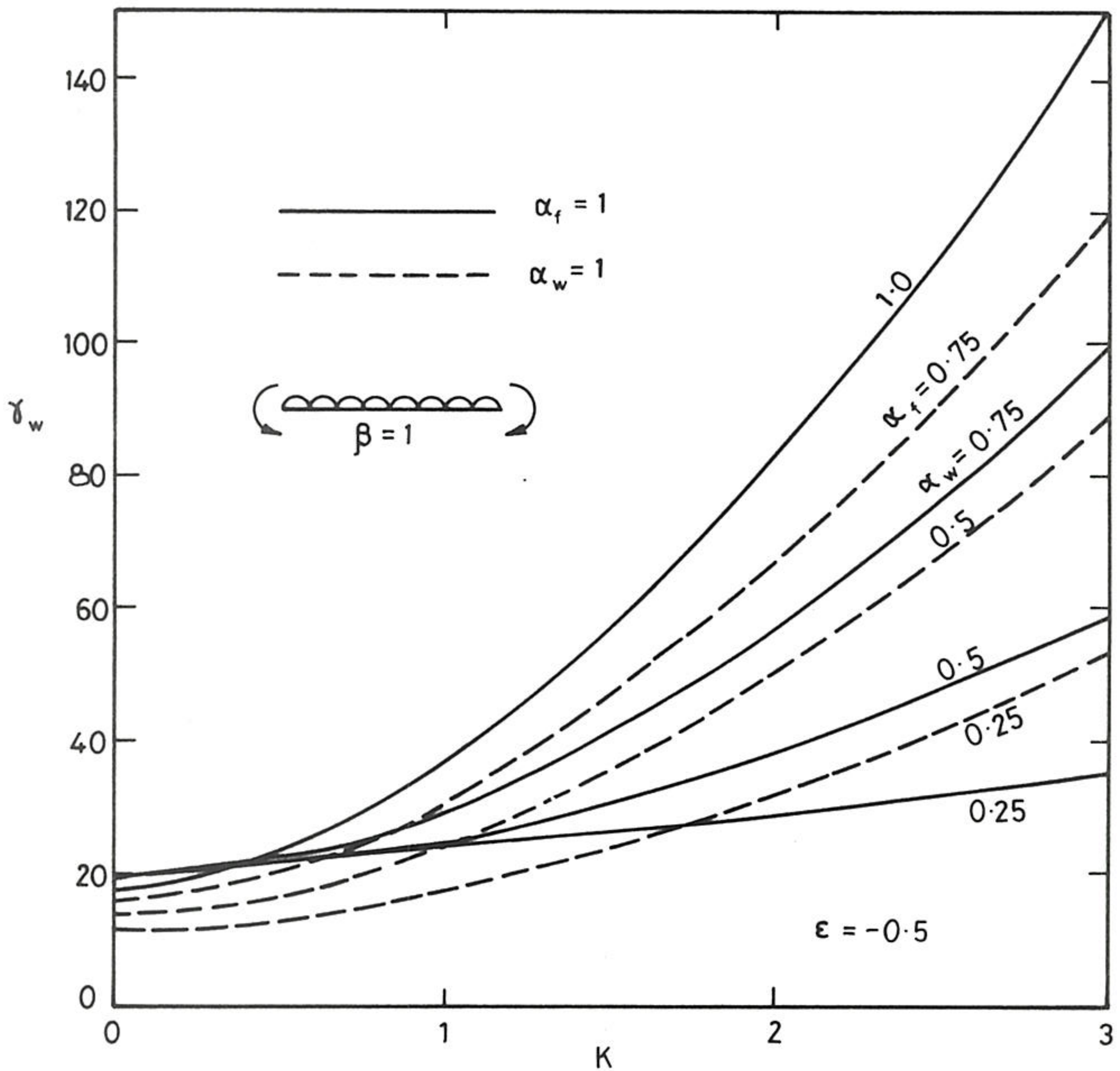


FIG 7 (c) B.I. BEAM WITH U.D.L. $\varepsilon = -0.5$

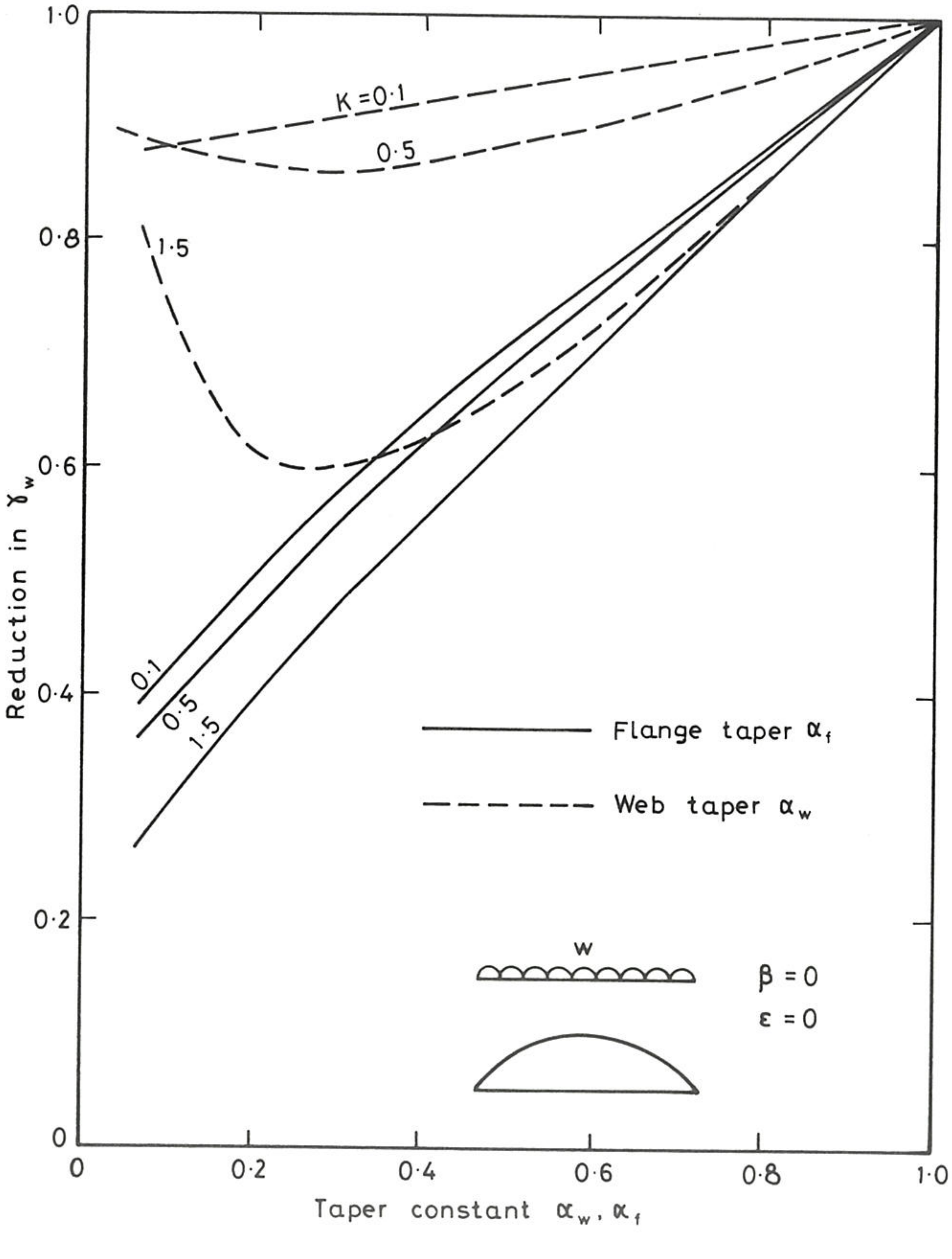
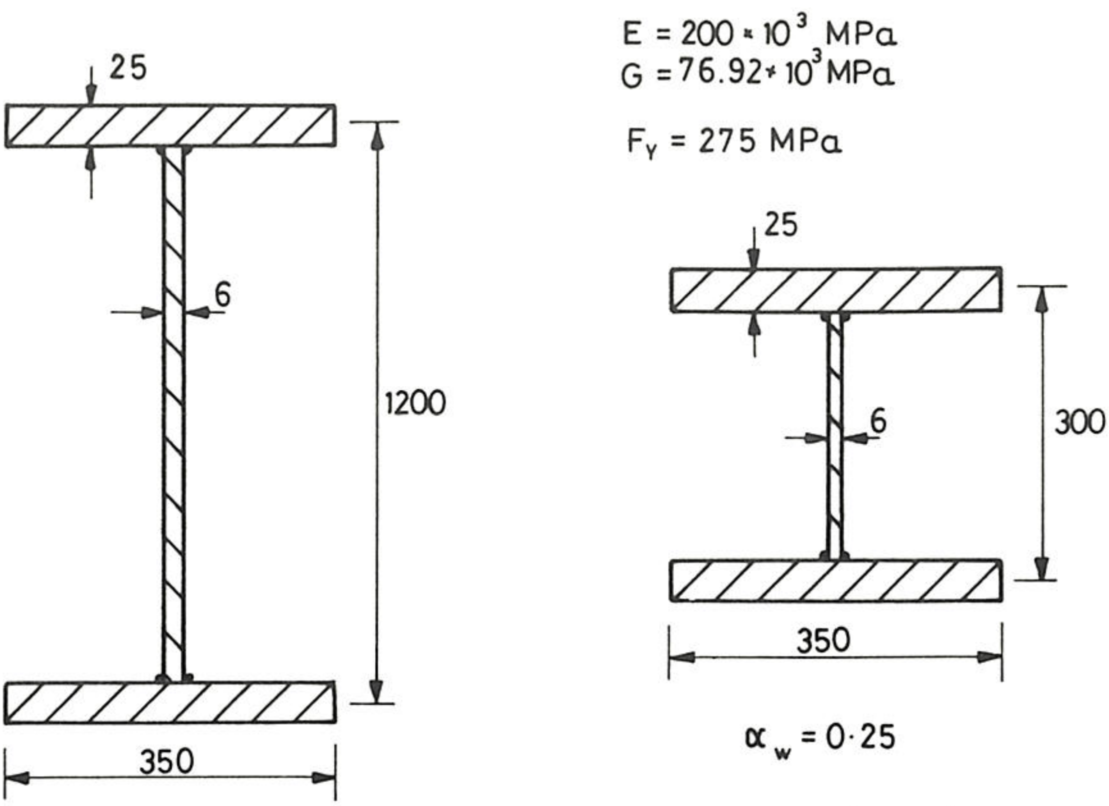


FIG. 8 REDUCTION IN ELASTIC BUCKLING LOAD DUE TO TAPERING



1.786×10^{8} mm^4	I_y	1.786×10^{8} mm^4
3.730×10^{6} mm^4	J	3.652×10^{6} mm^4
6.430×10^{13} mm^6	I_ω	4.019×10^{12} mm^6
2.455×10^{4} mm^2	A	1.915×10^{4} mm^2
85.3 mm	r_y	96.6 mm
3457 kNm	M_p	753 kNm

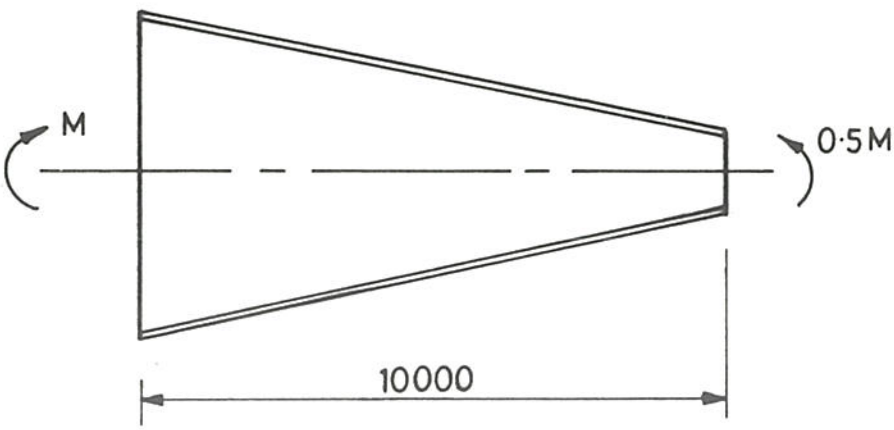


FIG. 9 DESIGN EXAMPLE